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Equilibrium, pseudoequilibrium and sliding-mode heteroclinic orbit in a Filippov-type plant disease model[☆]

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HIGHLIGHTS

- A Filippov-type plant disease model that incorporates cultural strategies and economic threshold policy is studied.
- To deal with the resulting non-smooth system, the generalized Lyapunov approach is employed.
- Our model is shown to admit stable equilibrium, unstable pseudoequilibrium and sliding-mode heteroclinic orbit.
- Biological implications in implementing control strategies for plant diseases are discussed.

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ABSTRACT

Plant diseases have caused tremendous crop losses and have massive impacts on food security and environment. Modeling the spread of plant diseases and understanding the dynamics of the resulting plant disease models may provide practical insights on designing effective control measures. In this paper, by incorporating cultural strategies and economic threshold policy, we present a Filippov-type plant disease model. The resulting model has state dependent discontinuous right-hand side and thus non-smooth analysis and generalized Lyapunov approach are employed for model analysis. We show that the model exhibits the phenomena of stable equilibrium, unstable pseudoequilibrium as well as sliding-mode heteroclinic orbit. Biological implications of our results in implementing control strategies for plant diseases are also discussed.

1. Introduction

Emerging infectious diseases caused by plant pathogens can cause tremendous crop losses leading to a global threat to food security and food safety [1,2]. Thus how to effectively control plant diseases has

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attracted much attention from scientists and various governments. Available control measures include biological, cultural, and chemical methods. Recently, the integrated disease management(IDM) that combines multiple control measures has been developed [3,4]. Due to the residues, the use of chemical control has a direct negative impact on environment and thus is not encouraged. To effectively control plant disease and to reduce the harm to environment, it is crucial to understand disease transmission dynamics. In this regard, mathematical modeling may play an important role and it has also been recognized as an important initiative to help design and evaluate control measures [5–15].

Bosch et al. [16] proposed a mathematical model of vegetatively propagated plant diseases with continuous control strategies. In practice, control activities usually occur periodically or impulsively, thus mathematical models with discontinuous control strategies have also been formulated, see, for example [10,17,18]. Since complete eradication of infected plants is very difficult, if not impossible, IDM allows tolerance threshold, called the economic threshold (ET), beyond which control measures are implemented to prevent the number of infected plants from exceeding the acceptable level. Comparing with the strategy of periodic removals, this threshold policy is more biologically and economically desirable, as control measures should be carried out only when the number of infected plants reaches the ET but not always periodically. Incorporating this threshold policy, some mathematical models are proposed to describe and analyze plant epidemics [7,18,19]. In these models, some are described by differential equations with discontinuous right-hand sides, called Filippov systems [20]. For Filippov systems, the analysis of dynamical behavior becomes more challenge due to the discontinuity of the right-hand side. Using the threshold policy and cultural control strategies, Zhao et al. [19] proposed a Lotka–Volterra Filippov-type plant disease model with a proportional planting rate, and investigated the global stability of five types of equilibria. For the model with a proportional rate in [19], it is significant to evaluate the effectiveness of the cultural control measures as the goal is to maintain infected plants not exceeding the given ET eventually. However, we find that the replanting and rouging measures have insignificant effect on the control goal. Note also the planting rate seems more reasonable to be a constant [18,21].

In this paper, we reconsider the plant disease model proposed in [19] but assuming a constant planting rate. The constant planting rate brings more difficulties in studying the dynamics of the resulting model, since the first integrals used in [19] no longer exist. Nevertheless, we establish the existence of equilibrium, pseudoequilibrium and sliding mode heteroclinic orbit and we fully describe the global dynamics of the resulting Filippov system. We show that the sliding-mode heteroclinic orbit connect a pseudoequilibrium to a real equilibrium or to another pseudoequilibrium. This reveals that the dynamics of Filippov systems is much more complex than that of the continuous ones. Our methods rely upon the theory for discontinuous differential equations and some results on non-smooth analysis from [20,22–24]. The generalized Lyapunov approach adopted in this paper allows us to drop the requirement on the smoothness of Lyapunov functions, which is needed in the literature [19,25]. From biological point of view, our results show that infected plants can be controlled not exceeding the given ET eventually by choosing appropriate replanting and rouging rates.

The rest of this paper is arranged as follows. In Section 2, we give a description about the model and present some preliminary results. Global dynamics of the model is investigated in Section 3. We devote Section 4 to biological implications of our results in controlling plant diseases. Finally, our findings are discussed in Section 5.

2. Model description and preliminaries

The plant disease model we consider in this paper is described by the following system

$$\begin{cases} \dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = A - \beta S(t)I(t) - \eta_1 S(t) + \psi(I)pS(t), \\ \dfrac{\mathrm{d}I(t)}{\mathrm{d}t} = \beta S(t)I(t) - \eta_2 I(t) - \psi(I)vI(t), \end{cases} \tag{2.1}$$