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On local strong solutions to the Cauchy problem of the two-dimensional full compressible magnetohydrodynamic equations with vacuum and zero heat conduction☆

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ABSTRACT

This paper concerns the Cauchy problem of the two-dimensional full compressible magnetohydrodynamic equations with zero heat-conduction and vacuum as far field density. In particular, the initial density can have compact support. We prove that the Cauchy problem admits a local strong solution provided both the initial density and the initial magnetic field decay not too slow at infinity.

1. Introduction and main results

We consider the two-dimensional full compressible magnetohydrodynamic (MHD) equations which read as follows:

$$\begin{cases} \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) + \nabla P = \mu \triangle u + (\mu + \lambda)\nabla(\operatorname{div} u) + H \cdot \nabla H - \dfrac{1}{2}\nabla |H|^2, \\ \dfrac{R}{\gamma - 1}\left((\rho\theta)_t + \operatorname{div}(\rho u \theta)\right) + P\operatorname{div} u = \kappa \Delta\theta + 2\mu|\mathfrak{D}(u)|^2 + \lambda(\operatorname{div} u)^2 + \nu|\nabla \times H|^2, \\ H_t - H \cdot \nabla u + u \cdot \nabla H + H\operatorname{div} u = \nu \Delta H, \quad \operatorname{div} H = 0. \end{cases} \tag{1.1}$$

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Here $t \geq 0$ is time, $x = (x_1, x_2) \in \Omega \subset \mathbb{R}^2$ is the spatial coordinate, $\rho = \rho(x,t)$, $u = (u^1, u^2)(x,t)$, $\theta = \theta(x,t)$ and $H = (H^1, H^2)(x,t)$ represent, respectively, the fluid density, velocity, absolute temperature and magnetic, and the pressure P is given by

$$P(\rho) = R\rho\theta, \quad (R > 0), \tag{1.2}$$

where R is a given constant. In addition, $\mathfrak{D}(u)$ is the deformation tensor

$$\mathfrak{D}(u) = \frac{1}{2}(\nabla u + (\nabla u)^{tr}).$$

The constant viscosity coefficients μ and λ satisfy the following hypothesis:

$$\mu > 0, \quad \mu + \lambda \geq 0. \tag{1.3}$$

The adiabatic constant γ and the heat conductivity coefficient κ are assumed to be

$$\gamma > 0, \qquad \kappa = 0. \tag{1.4}$$

Indeed, we consider the viscous compressible magnetohydrodynamic flows without heat-conduction ($\kappa = 0$), which implies that the energy equation $(1.1)_3$ can be rewritten equivalently as a hyperbolic equation for the pressure P as follows:

$$P_t + \mathrm{div}(Pu) + (\gamma - 1)P\mathrm{div}u = (\gamma - 1)Q(\nabla u) + \nu(\gamma - 1)|\nabla \times H|^2, \tag{1.5}$$

where

$$Q(\nabla u) \triangleq 2\mu|\mathfrak{D}(u)|^2 + \lambda(\mathrm{div}u)^2.$$

The constant $\nu > 0$ is the resistivity coefficient which is inversely proportional to the electrical conductivity constant and acts as the magnetic diffusivity of magnetic fields.

Let $\Omega = \mathbb{R}^2$, we consider the Cauchy problem (1.1)–(1.5), that is,

$$\begin{cases} \rho_t + \mathrm{div}(\rho u) = 0, \\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) + \nabla P = \mu \triangle u + (\mu + \lambda)\nabla(\mathrm{div}u) + H \cdot \nabla H - \frac{1}{2}\nabla|H|^2, \\ P_t + \mathrm{div}(Pu) + (\gamma - 1)P\mathrm{div}u = (\gamma - 1)Q(\nabla u) + \nu(\gamma - 1)|\nabla \times H|^2, \\ H_t - H \cdot \nabla u + u \cdot \nabla H + H\mathrm{div}u = \nu\Delta H, \quad \mathrm{div}H = 0, \end{cases} \tag{1.6}$$

with (ρ, u, P, H) vanishing at infinity (in some weak sense). For given initial data ρ_0, u_0, P_0 and H_0, we require that

$$\rho(x,0) = \rho_0(x), \qquad \rho u(x,0) = \rho_0 u_0(x), \qquad P(x,0) = P_0(x), \qquad H(x,0) = H_0(x), \quad x \in \mathbb{R}^2, \tag{1.7}$$

where $P_0 = R\rho_0\theta_0$ with $\theta_0(x) = \theta(x,0)$.

Magnetohydrodynamics concerns the motion of conducting fluids in an electromagnetic field and has a very broad range of applications, whose rigorous derivation from the compressible Navier–Stokes–Maxwell system has been proved by Kawashima–Shizuta [1,2] and Jiang–Li [3], respectively, for 1D(2D) cases and 3D one. There have been huge literatures on the study of the compressible MHD problem (1.1) by many physicists and mathematicians due to its physical importance, complexity, rich phenomena and mathematical challenges, see for example, [1–22] and the references therein. Now, we briefly recall some results concerned with the multi-dimensional compressible MHD equations which are more relatively with our problem. In the absence of vacuum, Kawashima [9] established the local and global well-posedness of the solutions to the compressible MHD equations, see also Vol'pert–Khudiaev [12] and Strohmer[10] for the local existence results. For the presence of vacuum, Fan–Yu [6] and Lü–Huang [13] established the local well-posedness of strong solutions to the 3D nonisentropic MHD flow and 2D isentropic case, respectively. Hu–Wang [7,8] and Fan–Yu [5] proved the global existence of renormalized solutions for large initial data. Concerning the