Nonlinear Analysis: Real World Applications 31 (2016) 580–592

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Expansion of a wedge of non-ideal gas into vacuum

M. Zafar*, V.D. Sharma

Department of Mathematics, Indian Institute of Technology Bombay, Mumbai-400076, India

ARTICLE INFO

Article history:
Received 19 September 2015
Received in revised form 1 March 2016
Accepted 10 March 2016
Available online 31 March 2016

Keywords:
Two-dimensional Euler equations
Van der Walls gas
Noble–Abel gas
Two-dimensional Riemann problem
Planar rarefaction waves
Invariant region

ABSTRACT

We study the problem of expansion of a wedge of non-ideal gas into vacuum in a two-dimensional bounded domain. The non-ideal gas is characterized by a van der Waals type equation of state. The problem is modeled by standard Euler equations of compressible flow, which are simplified by a transformation to similarity variables and then to hodograph transformation to arrive at a second order quasilinear partial differential equation in phase space; this, using Riemann variants, can be expressed as a non-homogeneous linearly degenerate system provided that the flow is supersonic. For the solution of the governing system, we study the interaction of two-dimensional planar rarefaction waves, which is a two-dimensional Riemann problem with piecewise constant data in the self-similar plane. The real gas effects, which significantly influence the flow regions and boundaries and which do not show-up in the ideal gas model, are elucidated; this aspect of the problem has not been considered until now.

1. Introduction

The problem of expansion of a wedge of ideal gas expanding into vacuum has received serious attention in recent years because of its relevance to a large number of physical processes relating to combustion engines, refrigerators, heat pumps, hot air balloons, gas storage, fire extinguishers and a lot of other practical applications, where the gases hardly behave like ideal gases and it becomes necessary to look for more realistic gases following a law that fits better with the behavior of gases in usage than the ideal gases (see [1,2]). It is well known that at high pressure or low temperature, the behavior of gases deviates from the ideal gas law and follows van der Waals type gas that deals with the possible real gas effects (without phase transition); examples cover a family of wave propagation problems with complicated interface patterns (see [3–8]). The van der Waals law takes into account the volume of molecules, which is important in many physical situations when the gas is compressed and fits better with the behavior of real gases than with that of a perfect gas. The reader is referred to an interesting piece of work, within the context of ideal gases, carried out by Li

* Corresponding author. Fax: +91 22 2572 3480.
E-mail addresses: zafar@math.iitb.ac.in (M. Zafar), vsharma@math.iitb.ac.in (V.D. Sharma).

http://dx.doi.org/10.1016/j.nonrwa.2016.03.006
1468-1218/

and Zheng [9–11], among several others (see [12–20] and the references cited there in), which has provided the prime motivation for the present work.

Indeed, there are mainly two approaches to get the solution of the gas expansion problem. One is classical hodograph transformation used by Li and Zheng in their pioneering work and the other one is the characteristic decomposition technique followed by Zheng et al. (see [17–19]). In his excellent and highly motivational work, Lai [21] has used characteristic decomposition technique to generalize the existence of the gas expansion problem for van der Waals gas. In the present study, we discuss the same problem in a non-ideal gas, referred to as the Noble–Abel gas, using classical hodograph transformation; the Noble–Abel gas is used in ballistic applications where, due to the high temperature, inter-molecular attraction energy is very small in comparison to the molecular kinetic energy. This, in turn, implies that the attraction term in van der Waals equation of state can be dropped without significant loss of accuracy (see [22,23]). Unlike [21], where a prior C^1-estimate of the solution to the problem of a wedge of van der Waals gas into vacuum is obtained by using the method of characteristic decomposition, we construct in this paper the self-similar solution to the interaction of planar rarefaction waves which lead in particular to the solution of the problem under consideration based on hodograph transformation. In addition, we have shown how the real gas effects, which manifest themselves through the van der Waals parameter b, influence the local structure of solution of the compressible Euler equations, namely, the flow variables, regions, boundaries, the shape of gas–vacuum interface, and estimates of the solution and its gradients.

Our paper is organized as follows. In Section 2, the problem is formulated and interaction of rarefaction waves influenced by the van der Waals parameter is discussed. In Section 3, the primary system and some properties of two-dimensional Euler system in a non-ideal gas are discussed. In Section 4, expansion of a wedge of a non-ideal gas into vacuum is discussed and boundary values influenced by the real gas effects are obtained. In Section 5, estimates of the solution and its gradients, influenced by non-ideal gas effects, are obtained.

2. Formulation of the problem

The basic equations of the present study are the Euler equations in two-dimensions which can be written as

$$\begin{aligned} &(\rho)_t + (\rho u)_x + (\rho v)_y = 0, \\ &(\rho u)_t + (p + \rho u^2)_x + (\rho u v)_y = 0, \\ &(\rho v)_t + (\rho u v)_x + (p + \rho v^2)_y = 0, \end{aligned} \tag{1}$$

where ρ is the gas density, (u, v) are the gas velocity components, and p is the pressure. Since the flow is isentropic, pressure p, density ρ, and the local sound speed $a = \sqrt{\frac{\partial p}{\partial \rho}}$ are related as (see [3])

$$p(\rho) = K\frac{\rho^\gamma}{(1 - b\rho)^\gamma}, \qquad a = \sqrt{\frac{\gamma p}{\rho(1 - b\rho)}}; \quad 0 \le b\rho < 1,\ K > 0. \tag{2}$$

Here $1 < \gamma < 3$ is the ratio of specific heats and b is the van der Waals excluded volume, which is assumed to be constant. A perfect gas can be seen as van der Waals gas with $b = 0$; presence of the co-volume b, representing the compressibility limit of the molecules, modifies non-trivially the analysis of Euler equations as the density in the model must be bounded. The equation of state in (2) can also be seen as a perfect gas polluted by dusty particles [24,25]. Consider the gas, whose initial state is one of constant density and zero velocity contained by two infinite walls l_1 and l_2 placed symmetrically with respect to x-axis making an angle 2θ (see Fig. 1(a)). When the walls l_1 and l_2 are removed at time $t = 0$, the wedge of non-ideal gas expands into a vacuum, and the gas particles in subsequent motion lie between the gas–vacuum interface and the rarefaction fronts propagating into the quiescent gas; the interface is, indeed, the zero