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Stability of a compressible two-fluid hyperbolic–elliptic system arising in fluid mechanics

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ABSTRACT

This paper deals with an initial–boundary value problem for the following one-dimensional two-fluid system

$$\begin{cases} n_t + (nu_g)_x = 0, & x \in I = (0,1),\ t > 0, \\ m_t + (mu_l)_x = 0, & \\ \alpha_g (P_g)_x = \mu_g (u_g)_{xx}, & \\ \alpha_l (P_l)_x = \mu_l (u_l)_{xx}, & \alpha_l + \alpha_g = 1, \end{cases}$$

where n and m represent, respectively, gas mass and liquid mass; u_g and u_l are corresponding fluid velocities whereas α_g and α_l are volume fractions occupied by the gas and liquid phase, and P_g and P_l are pressures associated with them. The model represents a submodel of the full two-fluid model studied in Bresch et al. (2012). An important difference between the model studied in the present work and that studied in Bresch et al. (2012) is that viscosity coefficients μ_l, μ_g are assumed to be constant. Bresch et al. assumed mass-dependent coefficients that allowed them to derive a so-called BD inequality which implies that masses are in H^1. Since we are excluded from following that route, we instead explore how the use of two non-equal pressure functions P_g and P_l (i.e., $P_l - P_g = f(m) \neq 0$) allows us to obtain global estimates that guarantee a stability result to hold. I.e., we prove that

$$m(\cdot,t) \to \widetilde{m}, \qquad n(\cdot,t) \to \widetilde{n}, \qquad u_l(\cdot,t), u_g(\cdot,t) \to 0, \quad \text{as } t \to \infty,$$

with respect to the norm in $L^\infty(I)$ for constant states $\widetilde{m}$ and $\widetilde{n}$. Estimates of the time asymptotic behavior are also provided.

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1. Introduction

This paper deals with a mathematical model for gas–liquid flow dynamics where the gas phase is modeled as an ideal gas whereas the liquid phase is assumed to be weakly compressible. The model is based on the so-called two-fluid formulation where the gas and liquid phase have separate mass and momentum conservation equations. In particular, the momentum equations involve a non-conservative pressure-related term, a viscous term and external force terms representing gravity and friction between fluid and wall as well as interfacial friction. The model takes the following form [1] (Chapter 10):

$$\begin{aligned} \partial_t(n) + \partial_x(nu_g) &= 0 \\ \partial_t(m) + \partial_x(mu_l) &= 0 \\ \partial_t(nu_g) + \partial_x(nu_g^2) + \alpha_g \partial_x P_g &= -f_g u_g - I(u_g - u_l) - ng + \partial_x(\mu_g \partial_x u_g) \\ \partial_t(mu_l) + \partial_x(mu_l^2) + \alpha_l \partial_x P_l &= -f_l u_l + I(u_g - u_l) - mg + \partial_x(\mu_l \partial_x u_l). \end{aligned} \tag{1.1}$$

Here

$$n = \alpha_g \rho_g \quad \text{and} \quad m = \alpha_l \rho_l,$$

where the volume fractions satisfy

$$\alpha_l + \alpha_g = 1, \tag{1.2}$$

whereas ρ_l, ρ_g are densities and u_l, u_g are fluid velocities associated with the liquid and gas phase. Moreover, the three first terms on the right hand side of the momentum equations represent, respectively, wall friction with coefficients f_g, f_l; interfacial friction with coefficient I; and gravity with gravity constant g. Finally, μ_g, μ_l are the viscosity coefficients.

Several challenges are associated with the model (1.1).

- The combination of different density–pressure laws corresponding to the different phases gives rise to non-conventional, nonlinear pressure functions that can be a challenge to deal with.
- Transition to single-phase regions, i.e, regions where m or n become zero, can happen because the volume fractions α_g, α_l become zero and/or because densities ρ_g, ρ_l vanish (formation of vacuum). Typically, we will need some uniform bounds on the masses m and n in order to derive higher order estimates.
- The non-conservative pressure terms $\alpha_g \partial_x P_g$ and $\alpha_l \partial_x P_l$ often prevent from applying arguments used for Navier–Stokes equations.

In [2] a model similar to (1.1) without external force terms is studied for two fluids described by density–pressure relations of the form

$$P_g = C_g \rho_g^{\gamma_g}, \qquad P_l = C_l \rho_l^{\gamma_l}, \quad \gamma_g, \gamma_l > 1, \; C_g, C_l > 0,$$

together with the assumption of equal pressure, i.e., $P_g = P_l = P$. A key assumption in their work is that the viscosity coefficients depend linearly on masses, i.e., are given as

$$\mu_g = \varepsilon_g n, \qquad \mu_l = \varepsilon_l m$$

for positive constants ε_g and ε_l. Thanks to this structure the model allows for deriving a so-called BD entropy estimate, which in turn ensures estimates of the form

$$\int ([m^{1/2}]_x^2 + [n^{1/2}]_x^2) \le C.$$

From these estimates the well-posedness is obtained as well as information about the long time behavior. We refer also to the recent work [3] which also largely relies on this approach but in a gas–liquid context