Nonlinear Analysis: Real World Applications 17 (2014) 71–88

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Reissner–Mindlin plate model with uncertain input data

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ARTICLE INFO

Article history:
Received 2 October 2012
Accepted 23 October 2013

ABSTRACT

A Reissner–Mindlin model of a plate resting on unilateral rigid piers and a unilateral elastic foundation is considered. Since the material coefficients of the orthotropic plate, stiffness of the foundation, and the lateral loading are uncertain, a method of the worst scenario (anti-optimization) is employed to find maximal values of some quantity of interest.

The state problem is formulated in terms of a variational inequality with a monotone operator. Using mixed-interpolated finite elements, approximations are proposed for the state problem and for the worst scenario problem. The solvability of the problems and a convergence of approximations is proved.

1. Introduction

An orthotropic elastic plate is given, the bending of which is described by means of the Reissner–Mindlin model. Two kinds of boundary conditions are considered: those of hard clamped or hard simply supported edges of the plate. The plate rests on a unilateral elastic foundation, being also supported unilaterally by several rigid piers. As a consequence, the state problem of the plate is represented by a variational inequality. Some input data, namely the material coefficients, the stiffness of the foundation, and the loading are *uncertain* being given in some compact sets of admissible values.

The model response is evaluated through a criterion functional (CF) (also known as a quantity of interest). Three variants are introduced. They are related to the deflection of the plate, to the intensity of shear stresses, and to the reactive forces acting on a pier, respectively.

The criterion functional value can vary due to uncertain inputs. The goal of the Worst (Case) Scenario Problem is to identify the maximum of the CF.

Although this approach (also known as anti-optimization, the term coined by I. Elishakoff in [1]) does not offer an insight into the behavior of the CF as deep as, for instance, probabilistic approaches, it is anchored in engineering reasoning about uncertainty and it is useful in situations where the knowledge about probabilistic uncertainty descriptors and parameters is limited or even unavailable [2,3], or where the knowledge of the worst-case situation is important. The latter can be observed if high reliability of structures is required [4,5].

Moreover, the worst scenario method is an integral part of other methods commonly used in uncertainty propagation analysis. This topic is further elaborated in Section 10.

In Section 2 we present the formulation of the state problem by means of a variational inequality with a monotone operator, which stems from the unilateral elastic foundation of Winkler's type. Sets of admissible (uncertain) input data are introduced in Section 3. We consider constant material coefficients, Lipschitz-continuous stiffness of the foundation and piecewise Lipschitz-continuous loading functions. The mixed-interpolated finite elements (see [6–8]) are employed to define approximate solutions of the state problem in Section 4.

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1468-1218/$ – see front matter
http://dx.doi.org/10.1016/j.nonrwa.2013.10.007

A general abstract theorem on two-parametric approximations of variational inequalities is proved in Section 5. We apply the theorem to the plate problem in Section 6, verifying all assumptions of the theorem. In Section 7 we define the Worst Scenario Problem. We introduce three criterion functionals and prove that they satisfy a continuity condition. In Section 8 we define an Approximate Worst Scenario Problem by means of solutions of the approximate state problem and prove its solvability. In Section 9 we prove the main convergence result: if the mesh-size tends to zero, then any sequence of solutions to the Approximate Worst Scenario Problem contains a subsequence that converges to a solution of the Worst Scenario Problem.

Related problems for an *optimal design* of anisotropic Reissner–Mindlin plates have been solved in [9].

Let us note that, throughout the paper, we will often use the symbol C as a general constant. This means that various constants can be denoted by the same symbol C in a chain of inequalities, for instance.

2. The State Problem

Let us assume that the midplane of the plate occupies a given bounded simply connected domain Ω with a polygonal boundary $\partial\Omega$.

We denote the standard Sobolev function spaces by $H^k(\Omega) \equiv W^{k,2}(\Omega)$, $k = 1, 2, \ldots$. Let the norm in $H^k(\Omega)$ be denoted by $\|\cdot\|_{k,2,\Omega}$ or, for brevity, $\|\cdot\|_{k,\Omega}$ or even $\|\cdot\|_k$, while $|\cdot|_{j,2,\Omega}$ be the jth seminorm. In this notation, $\|\cdot\|_{0,2,\Omega}$ stands for the L^2-norm. We use the notation

$$\partial_\alpha u \equiv \partial u/\partial x_\alpha, \qquad \beta \cdot \tau \equiv \tau_\alpha \beta_\alpha, \quad \alpha = 1, 2,$$

where a repeated Greek subscript implies the summation over the range 1, 2. Here and for now on, τ denotes the unit vector tangential to the boundary $\partial\Omega$ and β is a two-component vector.

The plate occupies the domain $\{x \equiv (x_1, x_2) \in \Omega,\ x_3 \in (-t, t)\}$, where $2t$ is the (constant) thickness of the plate.

The Reissner–Mindlin theory of elastic plates represents a refinement of the classical Kirchhoff theory of thin plates. Therefore it is suitable even for moderately thick plates. Whereas the Kirchhoff model assumes that the “in-plane” displacements u_1 and u_2 (in the directions of x_1 and x_2) have the form

$$u_\alpha(x_1, x_2, x_3) = -x_3 \partial w/\partial x_\alpha, \quad \alpha = 1, 2,$$

and the transversal displacement $u_3(x_1, x_2, x_3) = w(x_1, x_2)$ is x_3-independent (w is the deflection function), the Reissner–Mindlin model replaces the gradient ∇w by a vector β that is independent of w.

For a discussion of the model we refer to [7, Section VII.3].

Let $w \in H_0^1(\Omega)$ be the deflection function (positive in the direction of the x_3-axis) and let $\beta \in \mathcal{V}_1$ be the rotation vector function, where

$$\mathcal{V}_1 = \left\{\beta \in \left[H^1(\Omega)\right]^2 :\ \beta = 0 \text{ on } \Gamma_1 \text{ and } \beta \cdot \tau = 0 \text{ on } \Gamma_2 \equiv \partial\Omega \setminus \Gamma_1\right\}.$$

If $\Gamma_1 = \emptyset$, the plate is “hard simply supported”; if $\Gamma_2 = \emptyset$, the plate is “hard clamped” on the boundary.

We consider the small strain tensor components

$$\epsilon_{\alpha\nu} = -x_3\left(\partial_\nu \beta_\alpha + \partial_\alpha \beta_\nu\right)/2, \qquad 2\epsilon_{\alpha 3} = \partial_\alpha w - \beta_\alpha, \qquad \epsilon_{33} = 0, \quad \alpha, \nu = 1, 2.$$

To simplify the formulation of the stress–strain law, the entries of any symmetric 2×2 matrix $\{a_{ij}\}$ will be presented in the *vector notation* $\{a_k\}$, $k = 1, 2, 3$, where

$$a_k := a_{kk}, \quad k = 1, 2, \qquad a_3 := a_{12}.$$

Then instead of the standard stress–strain relations

$$\sigma_{\alpha\nu} = c_{\alpha\nu\gamma\delta}\epsilon_{\gamma\delta},$$

where $\alpha, \nu, \gamma, \delta \in \{1, 2\}$, we write

$$\sigma_i = \sum_{j=1}^{3} \mathcal{B}_{ij}\epsilon_j, \quad i = 1, 2, 3,$$

where $\{\mathcal{B}_{ij}\}$ is a symmetric 3×3 matrix.

We will consider homogeneous *orthotropic* materials, for which $\mathcal{B}_{i3} = 0,\ i = 1, 2$. Furthermore,

$$\sigma_{\alpha 3} = \mathcal{E}_\alpha \epsilon_{\alpha 3}, \quad \alpha = 1, 2, \qquad \sigma_{33} = 0,$$

where $\mathcal{E}_\alpha$ are given positive constants.

Remark 2.1. For *isotropic* materials,

$$\mathcal{B}_{11} = \mathcal{B}_{22} = \lambda + 2\mu, \qquad \mathcal{B}_{12} = \lambda, \qquad \mathcal{B}_{33} = 2\mu, \qquad \mathcal{E}_1 = \mathcal{E}_2 = 2k\mu,$$

where λ and μ are the Lamé constants and k is a constant *correction factor*. □