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Probabilistic stability analysis of social obesity epidemic by a delayed stochastic model

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ABSTRACT

Sufficient conditions for stability in probability of the equilibrium point of a social obesity epidemic model with distributed delay and stochastic perturbations are obtained. The obesity epidemic model is demonstrated on the example of the Region of Valencia, Spain. The considered nonlinear system is linearized in the neighborhood of the positive point of equilibrium and a sufficient condition for asymptotic mean square stability of the zero solution of the constructed linear system is obtained.

1. Introduction

Social obesity epidemic models are popular with researchers (see, for instance, [1–8]). In this paper the known nonlinear social obesity epidemic model [8] is generalized on the system with distributed delay. It is supposed also that this nonlinear system is exposed to additive stochastic perturbations of the type of the white noise that are directly proportional to the deviation of the system state from the equilibrium point. Such type of stochastic perturbations was first proposed in [9,10] and successfully used later in a many other works (see, for instance, [11–17]). The considered nonlinear system is linearized in the neighborhood of the positive point of equilibrium and a sufficient condition for asymptotic mean square stability of the zero solution of the constructed linear system is obtained. Since the order of nonlinearity is higher than 1 this condition is also a sufficient one [15,16] for stability in probability of the initial nonlinear system by stochastic perturbations.

This type of stability investigation was successfully used for investigation of different nonlinear mathematical models (SIR epidemic and some other medical models [9,13,14,18], predator–prey model [10,15–17], Nicholson blowfly model [15,16], inverted pendulum [15,16]) and can be an interesting tool, in particular, to policy makers.

1.1. Description of the considered model

To build the mathematical obesity model [8] the 24- to 65-year-old population is divided into three subpopulations based on their body mass index ($\text{BMI} = \text{Weight}/\text{Height}^2$). The classes or subpopulations are: individuals at a normal weight ($\text{BMI} < 25\ \text{kg/m}^2$) $N(t)$, people who are overweight ($25\ \text{kg/m}^2 \leq \text{BMI} < 30\ \text{kg/m}^2$) $S(t)$ and obese individuals ($\text{BMI} \geq 30\ \text{kg/m}^2$) $O(t)$.

The transitions between the different subpopulations are determined as follows: once an adult starts an unhealthy lifestyle he/she becomes addicted to the unhealthy lifestyle and starts a progression to being overweight $S(t)$ because of this lifestyle. If this adult continues with his/her unhealthy lifestyle he/she can become an obese individual $O(t)$. In both these classes individuals can stop his/her unhealthy lifestyle and then move to classes $N(t)$ and $S(t)$, respectively.

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The transitions between the subpopulations $N(t)$, $S(t)$ and $O(t)$ are governed by terms proportional to the sizes of these subpopulations. Conversely, the transitions from normal to overweight occurs through the transmission of an unhealthy lifestyle from the overweight and obese subpopulations to the normal-weight subpopulation, depending on the meetings among them. It is assumed that when an individual at a normal weight is *infected* by the transmission of an unhealthy lifestyle from the overweight and obese subpopulations, there is a time s during which the infection develops and it is only after that time that the infected individual (an individual at a normal weight with an unhealthy lifestyle) becomes an infectious individual (an overweight individual). This transit is modeled using the term

$$\beta N(t)\int_0^\infty (S(t-s)+O(t-s))dK(s),$$

where $K(s)$ is a nondecreasing function such that

$$\int_0^\infty dK(s)=1, \tag{1.1}$$

the integral being understood in the Stieltjes sense. The subpopulations' sizes and their behaviors with time determine the dynamic evolution of adulthood excess weight.

Thus, under the above assumptions, the following non-linear system of integro-differential equations is obtained:

$$\begin{aligned}
\dot{N}(t)&=\mu N_0-\mu N(t)-\beta N(t)\int_0^\infty (S(t-s)+O(t-s))dK(s)+\rho S(t),\\
\dot{S}(t)&=\mu S_0+\beta N(t)\int_0^\infty (S(t-s)+O(t-s))dK(s)-(\mu+\gamma+\rho)S(t)+\varepsilon O(t),\\
\dot{O}(t)&=\mu O_0+\gamma S(t)-(\mu+\varepsilon)O(t).
\end{aligned} \tag{1.2}$$

Remark 1.1. Note that the solution of the differential equation $\dot{x}(t)=a(t)-b(t)x(t)$, with $x(0)\geq 0$, $a(t)\geq 0$ and $b(t)\geq 0$, has the representation

$$x(t)=\left(x(0)+\int_0^t a(\tau)e^{\int_0^\tau b(s)ds}d\tau\right)e^{-\int_0^t b(s)ds}$$

and therefore $x(t)\geq 0$. Since each equation of (1.2) has the form of this equation then the system has non-negative solution.

The time-invariant parameters of this system of equations are:

- ε, the rate at which an obese adult with a healthy lifestyle becomes an overweight individual;
- μ, average stay time in the system of 24- to 65-year-old adults. Note that this parameter is not a birth rate and/or a death rate. In this case it is assumed as a recruitment and exit rate and its value is the same for entering and leaving the system and inversely proportional to the mean time spent by an adult in the system;
- ρ, the rate at which an overweight individual moves to the normal-weight subpopulation;
- β, transmission rate because of social pressure to adopt an unhealthy lifestyle (TV, friends, family, job and so on).
- γ, the rate at which an overweight 24- to 65-year-old adult becomes an obese individual because of unhealthy lifestyle;
- N_0, proportion of normal weight coming from the 23-year-old age group;
- S_0, proportion of overweight coming from the 23-year-old age group;
- O_0, proportion of obese coming from the 23-year-old age group.

Here the parameters ε, μ, ρ, β, γ are nonnegative numbers and N_0, S_0 O_0 are nonnegative numbers that satisfy the condition

$$N_0+S_0+O_0=1. \tag{1.3}$$

Taking into account these conditions and summing the equations of the system (1.2) and putting $Q(t)=N(t)+S(t)+O(t)$, $Q_0=N_0+S_0+O_0$, we obtain the equation $\dot{Q}(t)=-\mu(Q(t)-Q_0)$ with the initial condition $Q(0)=Q_0$. It is easy to see that this equation has the unique solution $Q(t)=Q_0$. So, from the assumption (1.3), i.e., $Q_0=1$, it follows that $Q(t)=1$ and the system (1.2) can be simplified to the system of two equations:

$$\begin{aligned}
\dot{N}(t)&=\mu N_0-\mu N(t)-\beta N(t)\int_0^\infty (1-N(t-s))dK(s)+\rho S(t),\\
\dot{S}(t)&=\mu S_0+\beta N(t)\int_0^\infty (1-N(t-s))dK(s)-(\mu+\gamma+\rho)S(t)+\varepsilon(1-N(t)-S(t)).
\end{aligned} \tag{1.4}$$