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New results on the positive almost periodic solutions for a model of hematopoiesis ☆

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ABSTRACT

This paper is concerned with a non-autonomous delayed model of hematopoiesis which is defined on the non-negative function space. Based on the definition of an almost periodic function, we employ a novel argument to establish a delay-independent criteria ensuring the existence, uniqueness, and global exponential stability of positive almost periodic solutions of the model with almost periodic coefficients and delays. Moreover, an example and its numerical simulation are given to illustrate the theoretical results.

1. Introduction

In the classic study of population dynamics, Mackey and Glass [1] proposed the following autonomous nonlinear delay differential equation:

$$p'(t) = -\gamma p(t) + \frac{\beta p^m(t-\tau)}{1+p^n(t-\tau)}, \quad \text{where } \gamma, \beta, n \in (0, +\infty),\ \tau, m \in [0, +\infty), \tag{1.1}$$

to describe some physiological control systems as their appropriate model (1.1) and its modifications have been now refereed to as a model of hematopoiesis (cell production), where $p(t)$ denotes the density of mature cells in blood circulation, the cells are lost from the circulation at a rate γ, the flux $f(p(t-\tau)) = \frac{\beta p^m(t-\tau)}{1+p^n(t-\tau)}$ of the cells into the circulation from the stem cell compartment depends on $p(t-\tau)$ at time $t-\tau$, and τ is the time delay between the production of immature cells in the bone marrow and their maturation for release in circulating bloodstreams. In the sense of the work of Mackey and Nechaeva [2], in (1.1), if $0 = m < n$, the feedback is negative, if $0 < m < n$, mixed, while $0 < m = n$, positive.

In the real-world phenomena, the variation of the environment (e.g., seasonal effects of weather, resource availability, reproduction, food supplies, mating habits, etc.) plays an important role. In the case, as pointed out in [3,4], the model (1.1) is naturally extended to the following nonautonomous nonlinear delay differential equation with time-varying coefficients and delays:

$$x'(t) = -a(t)x(t) + \sum_{i=1}^{K} \frac{b_i(t)x^m(t-\tau_i(t))}{1+x^n(t-\tau_i(t))}, \tag{1.2}$$

where $0 \le m \le n$, and

$$a,\ b_i,\ \tau_i : R \to (0, +\infty) \text{ are continuous functions for } i = 1, 2, \ldots, K.$$

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It is well known that the periodically varying environment and almost periodically varying environment are foundations for the theory of natural selection. Compared with periodic effects, almost periodic effects are more frequent (see [5,6]). Hence, the effects of the almost periodic environment on the evolutionary theory have been the object of intensive analysis by numerous authors and some of these results can be found in [7–10]. In particular, some criteria were established in [11–13] to ensure the existence and stability of positive almost periodic solutions of Eq. (1.2) with $m = 0$. Recently, the authors in [14,15] also obtained a new fixed point theorem to establish some sufficient conditions of the existence, nonexistence and uniqueness of a positive almost periodic solution for Eq. (1.2) with $m = 1$. However, to the best of our knowledge, there exist few results on the existence and exponential stability of the positive almost periodic solutions of (1.2) without $m = 0, 1$. On the other hand, since the exponential convergent rate can be unveiled, the global exponential stability of positive almost periodic solutions plays a key role in characterizing the behavior of a dynamical system (see [16–18]). Thus, it is worth while to continue to investigate the existence and global exponential stability of positive almost periodic solutions of (1.2) without $m = 0, 1$.

Motivated by the above discussions, in this paper, we consider the existence, uniqueness and global exponential stability of positive almost periodic solutions of (1.2). Here in this present paper, without assuming $m = 0, 1$, a new approach will be developed to obtain a delay-independent condition for the global exponential stability of the positive almost periodic solutions of (1.2), and the exponential convergent rate can be unveiled.

Throughout this paper, for $i = 1, 2, \ldots, K$, it will be assumed that $a, b_i, \tau_i : R \to (0, +\infty)$ are almost periodic functions, and

$$a^- = \inf_{t\in R} a(t), \qquad a^+ = \sup_{t\in R} a(t), \qquad b_i^- = \inf_{t\in R} b_i(t) > 0, \qquad b_i^+ = \sup_{t\in R} b_i(t), \tag{1.3}$$

$$0 \le m \le 1, \quad r = \max_{1\le i\le K} \left\{ \sup_{t\in R} \tau_i(t) \right\} > 0, \qquad \sum_{i=1}^{K} b_i^- > a^+. \tag{1.4}$$

Let R_+ denote nonnegative real number space, $C = C([-r, 0], R)$ be the continuous functions space equipped with the usual supremum norm $\|\cdot\|$, and let $C_+ = C([-r, 0], R_+)$. If $x(t)$ is defined on $[-r + t_0, \sigma)$ with $t_0, \sigma \in R$, then we define $x_t \in C$ where $x_t(\theta) = x(t + \theta)$ for all $\theta \in [-r, 0]$.

Due to the biological interpretation of model (1.2), only positive solutions are meaningful and therefore admissible. Thus we just consider admissible initial conditions

$$x_{t_0} = \varphi, \qquad \varphi \in C_+ \quad \text{and} \quad \varphi(0) > 0. \tag{1.5}$$

We write $x_t(t_0, \varphi)(x(t; t_0, \varphi))$ for an admissible solution of the admissible initial value problem (1.2) and (1.5). Also, let $[t_0, \eta(\varphi))$ be the maximal right-interval of existence of $x_t(t_0, \varphi)$.

We give the following remark, which will be of importance to the discussion of Lemma 2.2.

Remark 1.1. Let $f(u) = \frac{u^m}{1+u^n}$, one can get

$$\left.\begin{aligned} f'(u) &= \frac{u^{m-1}(m-(n-m)u^n)}{(1+u^n)^2} > 0, \quad \text{for all } u \in \left(0, \sqrt[n]{\frac{m}{n-m}}\right) \\ f'(u) &= \frac{u^{m-1}(m-(n-m)u^n)}{(1+u^n)^2} < 0, \quad \text{for all } u \in \left(\sqrt[n]{\frac{m}{n-m}}, +\infty\right) \end{aligned}\right\}, \quad \text{where } m < n. \tag{1.6}$$

Since

$$\lim_{\alpha\to 0^+} \frac{\alpha^{m-1}}{1+\alpha^n} = \begin{cases} 1, & m = 1, \\ +\infty, & m < 1, \end{cases}$$

we can choose a positive constant κ such that

$$\frac{\alpha^{m-1}}{1+\alpha^n} > \frac{a^+}{\sum_{i=1}^{K} b_i^-}, \quad \text{for all } \alpha \in (0, \kappa],$$

and

$$\kappa < \sqrt[n]{\frac{m}{n-m}}, \quad \text{if } m < n. \tag{1.7}$$

Moreover, from (1.6), (1.7) implies there exists a constant $\tilde{\kappa}$ such that

$$\kappa < \sqrt[n]{\frac{m}{n-m}} < \tilde{\kappa}, \qquad \frac{\kappa^m}{1+\kappa^n} = \frac{\tilde{\kappa}^m}{1+\tilde{\kappa}^n}, \quad \text{if } m < n. \tag{1.8}$$