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Calderón–Zygmund estimates for a class of quasilinear parabolic equations

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ABSTRACT

In this paper we obtain the following local Calderón–Zygmund estimates

$$G\left(|\mathbf{f}|\right) \in L^q_{loc}(\Omega_T) \Rightarrow G\left(|\nabla u|\right) \in L^q_{loc}(\Omega_T) \quad \text{for any} \quad q \geq 1$$

of weak solutions for a class of quasilinear parabolic equations

$$u_t - \text{div}\ \left(a\left(|\nabla u|\right)\nabla u\right) = \text{div}\ \left(a\left(|\mathbf{f}|\right)\mathbf{f}\right) \quad \text{in} \quad \Omega_T,$$

where $G(t) = \int_0^t \tau a(\tau)\ d\tau$ for $t \geq 0$. We remark that

$$G(t) = |t|^p \log\Big(1+|t|\Big) \quad \text{for} \quad p > 2$$

satisfies the given conditions in this work. Moreover, we would like to point out that our results improve the known results for such equations.

1. Introduction

In this paper we are concerned with the local L^p-type regularity estimates of weak solutions for the following quasilinear parabolic equations

$$u_t - \text{div}\left(a\left(|\nabla u|\right)\nabla u\right) = \text{div}\left(a\left(|\mathbf{f}|\right)\mathbf{f}\right) \qquad \text{in } \Omega_T = \Omega \times (0,T), \tag{1.1}$$

where Ω is an open bounded domain in $\mathbb{R}^n$ and the function $a : (0,\infty) \to (0,\infty) \in C^1(0,\infty)$ satisfies

$$0 \leq i_a =: \inf_{t>0} \frac{ta'(t)}{a(t)} \leq \sup_{t>0} \frac{ta'(t)}{a(t)} =: s_a < \infty. \tag{1.2}$$

Especially when $a(t) = t^{p-2}$ and then $p = s_a + 2 = i_a + 2$, (1.1) is reduced to the parabolic p-Laplace equation

$$u_t - \text{div}\left(|\nabla u|^{p-2}\nabla u\right) = \text{div}\left(|\mathbf{f}|^{p-2}\mathbf{f}\right) \qquad \text{in } \Omega_T. \tag{1.3}$$

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Now we denote

$$g(t) = ta(t) \tag{1.4}$$

and

$$G(t) = \int_0^t \tau a(\tau) d\tau = \int_0^t g(\tau) d\tau \quad \text{for} \quad t \geq 0. \tag{1.5}$$

Then from (1.2) it is easy to check that

$$g(t) \text{ is strictly increasing and continuous over } [0, +\infty) \tag{1.6}$$

and

$$G(t) \text{ is convex}, G(0) = 0 \text{ and increasing over } [0, +\infty). \tag{1.7}$$

L^p-type regularity is the fundamental theory of partial differential equations, which plays an important role in the theory of elliptic and parabolic equations, and is the basis for the existence and uniqueness of solutions. L^p estimates for the second order elliptic and parabolic problems have been obtained by different techniques. Many authors (see [9,11,17–19,22,24,25,27,29,30]) have studied L^q $(q \geq p)$ estimates of the gradient of weak solutions for

$$\text{div}\left(|\nabla u|^{p-2}\nabla u\right) = \text{div}\left(|\mathbf{f}|^{p-2}\mathbf{f}\right) \quad \text{in } \Omega \tag{1.8}$$

and the general case with different assumptions on the coefficients and domains. Recently, Cianchi and Maz'ya [14,15] proved global Lipschitz regularity for the Dirichlet and Neumann elliptic boundary value problems of the form

$$\text{div}\left(a\left(|\nabla u|\right)\nabla u\right) = f \quad \text{in } \Omega. \tag{1.9}$$

Moreover, Cianchi and Maz'ya [16] obtained a sharp estimate for the decreasing rearrangement of the length of the gradient for the Dirichlet and Neumann elliptic boundary value problems of (1.9).

Different from the elliptic case (1.8), (1.3) is not homogeneous even if $\mathbf{f} \equiv 0$, which is one of the most difficulties (see [6]). Kinnunen and Lewis [23] obtained a reverse Hölder inequality of the gradient for weak solutions of (1.3) and the general case. Furthermore, Acerbi and Mingione [1] obtained the following estimates in Sobolev spaces

$$|\mathbf{f}|^p \in L^q_{loc}(\Omega) \Rightarrow |\nabla u|^p \in L^q_{loc}(\Omega) \quad \text{for any} \quad q \geq 1 \tag{1.10}$$

with

$$\int_{Q_r} |\nabla u|^{pq} dz \leq C\left[\left(\int_{Q_{2r}} |\nabla u|^p dz\right)^q + \int_{Q_{2r}} |\mathbf{f}|^{pq} + 1 dz\right]^{p/2}, \tag{1.11}$$

where $Q_{2r} = B_{2r} \times (-4r^2, 4r^2] \subset \Omega_T$, for weak solutions of (1.3) and the general case. Moreover, many authors [4,7,20,28] proved Lipschitz regularity, Caccioppoli-type estimate and existence of weak solutions for (1.1), respectively. The purpose of this paper is to extend (1.10) and (1.11) for weak solutions of (1.1). In particular, we are interested in the following local Calderón–Zygmund estimates like

$$G\left(|\mathbf{f}|\right) \in L^q_{loc}(\Omega_T) \Rightarrow G\left(|\nabla u|\right) \in L^q_{loc}(\Omega_T) \quad \text{for any} \quad q \geq 1 \tag{1.12}$$