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Nonlocal nonlinear reaction preventing blow-up in supercritical case of chemotaxis system

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A B S T R A C T

This paper is devoted to the analysis of non-negative solutions for the chemotaxis model with nonlocal nonlinear source in bounded domain. The qualitative behavior of solutions is determined by the nonlinearity from the aggregation and the reaction. When the growth factor is stronger than the dampening effect, with the help of the nonlocal nonlinear term in the reaction, for appropriately chosen exponents and arbitrary initial data, the model admits a classical solution which is uniformly bounded. Moreover, when the growth factor has the same order with the dampening effect, the nonlocal nonlinear exponents can prevent the chemotactic collapse.

1. Introduction

The Keller–Segel model in Chemotaxis was originally introduced by Keller and Segel [13,14] to describe the characteristic movement of cells, the cells can move toward the increasing signal concentration or can be repulsive by the signal concentration. From then on, mathematical models to describe chemotaxis have been widely proposed in the last few years. The simplest version contains the competition among the diffusion, reproduction and the nonlocal aggregation satisfying [21]

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u^\sigma \nabla c) + f(u), & x \in \Omega, t > 0, \\ \tau c_t - \Delta c + c = u^\xi, & x \in \Omega, t > 0, \\ u(x,0) = u_0(x) \geq 0, & x \in \Omega. \end{cases} \tag{1.1}$$

In the modeling, Ω is either a bounded domain in $\mathbb{R}^n$ or the whole space. In the context of biological aggregation, $u(x,t)$ represents the bacteria density, $c(x,t)$ is the chemical substance concentration. The reaction term describes the reproduction rate of the bacteria where the resources of the environment can be consumed either locally or nonlocally. When chemicals diffuse much faster than cells [12], (1.1) can be

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reduced into parabolic–elliptic model, i.e.

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u^\sigma\nabla c) + f(u), & x\in\Omega, t>0, \\ -\Delta c + c = u^\xi, & x\in\Omega, t>0. \end{cases} \tag{1.2}$$

In the following we will report some of the related previous results on (1.2) in terms of $f(u)$.

(1.2) with $f(u)\equiv 0$ is the classical Keller–Segel model which expresses the random movement (brownian motion) of the cells with a bias directed by the chemoattractant concentration [21]. This system has been widely studied, such as [7,12,18] and the references therein. It is proved that for the following problem with Neumann boundary condition

$$\begin{aligned} &u_t = \Delta u - \chi\nabla\cdot(u\nabla c), \\ &-\Delta c = u - 1, \end{aligned}$$

blow-up never occurs in one dimension [18]. While in two dimensions, there exists a threshold number for the initial data that can separate global existence and finite time blow-up [12].

When $f(u)\neq 0$, the logistic growth including the consumption of resources around the environment is taken into account in chemotaxis models. There are quite a number of works handling such type of model with logistic growth describes the situation where the influence of nonlocal terms is omitted. Here we can only list some of the results which are closely related to our model.

For $\sigma=\xi=1$, the authors in [23] proved that model (1.2) with

$$f(u)\le a - bu^2, \quad u\ge 0 \tag{1.3}$$

possesses a global bounded classical solution for either $n\le 2$ or $n\ge 3$ and $b>\frac{n-2}{2}\chi$. In addition, for all $n\ge 1, b>0$ and arbitrary initial data there exists at least one global weak solution given by $f(u)\ge -c_0(u^2+1),\ \forall u>0$ with some $c_0>0$.

For more general case, in [26] the authors considered the model

$$\begin{aligned} &u_t = \nabla\cdot(D(u)\nabla u) - \chi\nabla\cdot(u\nabla c) + f(u), \\ &-\Delta c + c = u. \end{aligned}$$

Here $f(u)$ is smooth satisfies $f(0)\ge 0$ and

$$f(u)\le a - bu^\gamma$$

for all $u\ge 0$ with $a\ge 0, b>0$ and $\gamma>1$. $D(u)\in C^2([0,\infty))$ and there exist some constants $c_D>0$ and $m\ge 1$ such that $D(u)\ge c_D u^{m-1}$ for all $u>0$ as well as $D(u)>0$ for all $u\ge 0$. They proved that if $\gamma\ge 2$ and $b>b^*$ where

$$b^* = \begin{cases} \dfrac{(2-m)n-2}{(2-m)n}\chi, & if \quad m<2-2/n, \\ 0, & if \quad m\ge 2-2/n, \end{cases}$$

or $\gamma\in(1,2)$ and $m>2-2/n$, then the model has a unique nonnegative classical solution which is global and bounded.

In [4] the authors considered

$$\begin{aligned} &u_t = \Delta u - \chi\nabla\cdot(u^\sigma\nabla c) + \mu u(1-u^\alpha), \\ &-\Delta c + c = u^\xi \end{aligned}$$