Nonlinear Analysis 174 (2018) 242–277

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Nonlinear stability of rarefaction waves for the compressible Navier–Stokes equations with zero heat conductivity

Jiayi Hu[a,b], Hui Yin[c,d,*]

[a] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China
[b] School of Mathematics and Statistics, South-Central University for Nationalities, Wuhan 430074, China
[c] School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, China
[d] Hubei Key Laboratory of Engineering Modeling and Scientific Computing, Huazhong University of Science and Technology, Wuhan 430074, China

ARTICLE INFO

Article history:
Received 11 March 2018
Accepted 24 April 2018
Communicated by Enzo Mitidieri

MSC:
35L65
35L60

Keywords:
The compressible Navier–Stokes equations
Zero heat conductivity
Rarefaction waves
Nonlinear stability

ABSTRACT

This paper is concerned with the time-asymptotic nonlinear stability of rarefaction waves to the Cauchy problem of the one-dimensional compressible Navier–Stokes equations with zero heat conductivity. Under the assumption that the unique global entropy solution to the resulting Riemann problem of the corresponding compressible Euler equations consists of rarefaction waves only, then if both the initial perturbation and the strengths of rarefaction waves are assumed to be suitably small, we show that its Cauchy problem admits a unique global solution which tends time-asymptotically toward the rarefaction waves. This result is proved by using the elementary energy method and the argument developed by Kawashima and Okada (1982).

1. Introduction and main results

Consider the one-dimensional compressible Navier–Stokes equations in the Lagrangian coordinates

$$\begin{aligned} v_t - u_x &= 0, \\ u_t + p_x &= \left(\frac{\mu u_x}{v}\right)_x, \\ \left(e + \frac{u^2}{2}\right)_t + (up)_x &= \left(\frac{\kappa\theta_x}{v} + \frac{\mu u u_x}{v}\right)_x, \end{aligned} \tag{1.1}$$

where the unknowns $v > 0, u, \theta > 0, p > 0, e$, and s represent the specific volume, the velocity, the absolute temperature, the pressure, the internal energy, and the entropy of the gas, respectively. The coefficients of

* Corresponding author at: School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, China.

E-mail addresses: jiayihu@whu.edu.cn (J. Hu), yinhui928@126.com (H. Yin).

https://doi.org/10.1016/j.na.2018.04.026

viscosity and heat-conductivity, μ and κ, are positive constants or identically zero. We assume, as usual in thermodynamics, that by using any given two of the five thermodynamical variables, v, θ, p, e, and s, the remaining three variables are their functions.

The second law of thermodynamics asserts that

$$\theta ds = de + pdv.$$

From which, if we choose (v,θ), (v,s), or (v,e) as independent variables and write $(p,e,s) = (p(v,\theta), e(v,\theta), s(v,\theta))$, or $(p,e,\theta) = (\tilde{p}(v,s), \tilde{e}(v,s), \tilde{\theta}(v,s))$, or $(p,s,\theta) = (\hat{p}(v,e), \hat{s}(v,e), \hat{\theta}(v,e))$ respectively, then we can deduce that

$$\begin{aligned} s_v(v,\theta) &= p_\theta(v,\theta),\\ s_\theta(v,\theta) &= \frac{e_\theta(v,\theta)}{\theta},\\ e_v(v,\theta) &= \theta p_\theta(v,\theta) - p(v,\theta), \end{aligned} \tag{1.2}$$

$$\begin{aligned} \tilde{e}_v(v,s) &= -p(v,\theta),\\ \tilde{e}_s(v,s) &= \theta,\\ \tilde{p}_v(v,s) &= p_v(v,\theta) - \frac{\theta(p_\theta(v,\theta))^2}{e_\theta(v,\theta)},\\ \tilde{p}_s(v,s) &= \frac{\theta p_\theta(v,\theta)}{e_\theta(v,\theta)},\\ \tilde{\theta}_v(v,s) &= -\frac{\theta p_\theta(v,\theta)}{e_\theta(v,\theta)},\\ \tilde{\theta}_s(v,s) &= \frac{\theta}{e_\theta(v,\theta)}, \end{aligned} \tag{1.3}$$

or

$$\begin{aligned} \hat{s}_e(v,e) &= \frac{1}{\theta},\\ \hat{s}_v(v,e) &= \frac{p(v,\theta)}{\theta},\\ \hat{p}_e(v,e) &= \frac{p_\theta(v,\theta)}{e_\theta(v,\theta)},\\ \hat{p}_v(v,e) &= \left(p_v(v,\theta) - \frac{\theta(p_\theta(v,\theta))^2}{e_\theta(v,\theta)}\right) + \frac{p(v,\theta)p_\theta(v,\theta)}{e_\theta(v,\theta)},\\ \hat{\theta}_e(v,e) &= \frac{1}{e_\theta(v,\theta)},\\ \hat{\theta}_v(v,e) &= \frac{p(v,\theta) - \theta p_\theta(v,\theta)}{e_\theta(v,\theta)}. \end{aligned} \tag{1.4}$$

Throughout this paper, the pressure function $p(v,\theta)$ and the internal energy $e(v,\theta)$ are assumed to satisfy

$$(\mathrm{H}_1) \qquad p_v(v,\theta) = \frac{\partial p(v,\theta)}{\partial v} < 0, \quad e_\theta(v,\theta) = \frac{\partial e(v,\theta)}{\partial \theta} > 0$$

and

$$(\mathrm{H}_2) \qquad \tilde{p}_{vv}(v,s) = \frac{\partial^2 \tilde{p}(v,s)}{\partial v^2} > 0 \ \text{ and } \ \tilde{p}(v,s) \ \text{ is convex with respect to} (v,s).$$

From (1.3)$_2$ and (H$_1$), we have

$$\tilde{p}_v(v,s) = p_v(v,\theta) - \frac{\theta(p_\theta(v,\theta))^2}{e_\theta(v,\theta)} < 0, \tag{1.5}$$

$$\begin{aligned} \tilde{e}_{ss}(v,s) &= \frac{\theta}{e_\theta(v,\theta)} > 0,\\ \tilde{e}_{vs}(v,s) &= -\frac{\theta p_\theta(v,\theta)}{e_\theta(v,\theta)},\\ \tilde{e}_{vv}(v,s) &= -p_v(v,\theta) + \frac{\theta(p_\theta(v,\theta))^2}{e_\theta(v,\theta)} > 0, \end{aligned} \tag{1.6}$$