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Solitary water waves propagating on an underlying uniform current over a finite depth

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A B S T R A C T

We investigate the nature of velocity and pressure fields for solitary water waves (without any restriction on amplitude) propagating over finite depth with uniform underlying currents. We show that two cases exist corresponding to the fluid velocities u greater than or less than the solitary wave propagation speed c, and these two cases result in different nature of flow fields. We prove the conditions for existence of solitary waves in 2D with uniform underlying current over finite depth for these two cases. We also derive some bounds on Froude number and maximum amplitude of solitary wave height.

1. Introduction

Originally discovered by Scott Russell from experiments carried out in canals, solitary waves are observed in nature and have been simulated in laboratory. Since its first discovery, solitary waves have attracted the attention of the researchers and a large volume of research has been carried out including some recent revelations about these waves (Constantin and Escher (2007) [10], Constantin et al. (2011) [11]). Solitary waves are gravity waves as the only restoring force acting on the water in this case is gravity (since surface tension can be ignored if amplitudes are not very small). Solitary waves have a symmetrical profile and propagates on the water surface as a single hump decaying rapidly on either side to almost flat level far ahead and far away from the crest. These waves have a one-dimensional structure as no localized steady gravity waves with a two-dimensional profile (i.e., 3D solitary waves) can exist (Craig (2002), [13]).

The rigorous theory of solitary waves was developed by Friedrichs and Hyers (1954) [16] and was further addressed by Beale (1977) [5]. A significant contribution to the analytical theory for solitary waves has been made by Toland and collaborators [3,2,1] which has provided information about the structure of these waves.

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Even though a large amount of investigation has been carried out on the solitary waves, properties of the underlying flow has been the attention of research in the recent past with only a few papers concentrating on this aspect. A good survey of the literature is available in [8]. Constantin and Escher (2007) [10] studied the particle trajectories in solitary waves while Constantin et al. (2011) [11] developed the mathematical theory for the pressure beneath a solitary wave including experimental verification of the same. The rigorous analysis of the pressure field in its own right for water waves was initiated in [12], and some interesting further recent developments can be found in the papers [9,17,20,21]. Some studies have focussed on linearized theories for small amplitudes (Escher and Schlurmann (2008) [14], Tasi et al. (2015) [23]) and on numerical investigations (Evans and Ford (1996) [15]). Focus on large amplitude waves has been recent and has been concentrated on by researchers such as Chen et al. (2009) [6], Constantin (2006) [7], Constantin and Strauss (2010) [12] and Hsu et al. (2009) [19]. Wheeler (2013, 2015) [24,25] constructed large amplitude solutions for solitary waves with vorticity and provided some bounds on Froude number. Henry (2013) [18] has developed the pressure transfer function for solitary water waves in the presence of vorticity. Existing literature on solitary waves indicates that in spite of the large volume of research activities in the area, investigations focussing on the flow and pressure fields beneath the water surface for solitary waves of large amplitude over finite depth (including underlying current) are still incomplete. The approach in this paper to study the velocity and pressure field beneath the flow is motivated by the considerations in nonlinear steady periodic Stokes wave of large amplitude (Constantin and Strauss (2010) [12]), and the results on existence are influenced by the work of Amick and Toland [2]. Recently, Basu (2016) [4] has reported some results on two dimensional irrotational free surface water waves with underlying current and these results also contribute to the proofs and derivations in this paper.

The aim of this paper is to investigate solitary waves of large amplitude propagating on uniform current over a flat bed. Some results on velocity and pressure beneath the flow are derived. The existence results for solitary water waves with uniform current have been proved and bounds on Froude number have been established. Finally, bounds on the maximum amplitude of a solitary water wave have also been derived.

2. Preliminaries and fundamental equations

We consider a 2D flow described using a Cartesian coordinate system (X, Y). The depth of the fluid medium at rest is $d > 0$ and the flat bed is given by $Y = -d$. The free surface of the water is denoted by $Y = \eta(X - ct)$, where $c > 0$ is the constant speed of the travelling solitary wave.

The governing equations of motion are described by Euler equations

$$\begin{aligned} u_t + uu_X + vu_Y &= -P_X \\ v_t + uv_X + vv_Y &= -P_Y - g \end{aligned} \tag{1}$$

where $P(X - ct, Y)$ is the hydrodynamic pressure and g is the acceleration due to gravity. The boundary conditions are

$$v = \eta_t + u\eta_X \quad \text{on} \quad Y = \eta(X - ct) \tag{2}$$

and

$$P = P_{atm} \quad \text{on} \quad Y = \eta(X - ct) \tag{3}$$

where P_{atm} is the constant atmospheric pressure.

The flat bed is impermeable, i.e.

$$v = 0 \quad \text{on} \quad Y = -d\ . \tag{4}$$