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Bifurcation results for a class of prescribed mean curvature equations in bounded domains

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ABSTRACT

Bifurcation for prescribed mean curvature equations in one dimension has been intensively investigated in recent years, and a striking phenomenon discovered is that the length of the interval may affect the shapes of bifurcation curves. However, to our best knowledge, no such results are known in higher dimensions. In this paper, we study a class of prescribed mean curvature equations in bounded domains of $\mathbb{R}^N$ with general nonlinearities f satisfying $f(0) = 0$ and $f'(0) > 0$. We establish some formulas for directions of bifurcation at simple eigenvalues, which lead to a sufficient and necessary condition to ensure that the directions depend on the size of the domain. In contrast, this phenomenon does not occur for the semilinear case. Some interesting examples, including logistic and perturbed exponential nonlinearities, are also investigated.

1. Introduction

Consider the prescribed mean curvature problem

$$\begin{cases} -\operatorname{div}\left(\dfrac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = \lambda f(u), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$ $(N \geqslant 1)$ and λ is a positive parameter. We are concerned with the effect of the size of the domain on the shape of bifurcation curve.

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Bifurcation for the semilinear problem

$$\begin{cases} -\Delta u = \lambda f(u), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

has been well studied; see e.g., the nice surveys in [18,21] and the recent monograph [13]. Different from Δ, the mean curvature operator $u \mapsto \mathrm{div}(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}})$ is nonlinear and nonhomogeneous. This leads to that the quasilinear problem (1.1) is more difficult to handle than the semilinear problem (1.2). Bifurcation for the prescribed mean curvature equation is still far from being well understood; see [19] for $f(u) = u$, [6,30] for $\Omega = \mathbb{R}^N$, and [15,16] for solutions in the bounded variation space.

Recently, one-dimensional case of (1.1) has received considerable attention and various results on existence and exact multiplicity have been obtained (see e.g., [1–5,7–9,11,12,14,17,19,20,22–28,31,32]). Among them, a striking phenomenon discovered is that the length of the interval may affect on the shapes of bifurcation curves [3,4,7–9,12,14,22–28]. However, to our best knowledge, no such results are known in higher-dimension cases of (1.1). Our research is inspired by [4,8]. In [4], the authors investigate the case $\Omega = (0, L)$ and $f(u) = u - u^3$ with Neumann boundary conditions, and apply the Liapunov–Schmidt reduction to prove that at each eigenvalue $\lambda_n = (\frac{n\pi}{L})^2$, a supercritical pitchfork occurs if $L > \frac{n\pi}{\sqrt{2}}$ and a subcritical pitchfork occurs if $L < \frac{n\pi}{\sqrt{2}}$. In [8], the authors use a time-map method to establish bifurcation diagrams for positive solution of (1.1) with $f(u) = e^{\frac{au}{a+u}} - 1$ and $\Omega = (-L, L)$; the bifurcation diagrams derived imply that when $a = 2$, the length L may affect the direction of local bifurcation. The motivation of this paper is to generalize these one-dimensional results from special examples to more general f and to higher dimensions.

In the present paper, we study (1.1) with general nonlinearities f satisfying $f(0) = 0$ and $f'(0) > 0$. By establishing and analyzing formulas for directions of bifurcation at simple eigenvalues, we obtain a sufficient and necessary condition to ensure that the direction of bifurcation at the principal eigenvalue depends on the size of the domain. In contrast, this phenomenon does not occur for the corresponding semilinear problem. Some interesting examples are also discussed. A key point is that although the linearization of the mean curvature operator at the trivial solution is the Laplace operator, the higher order terms play a crucial role in determining the directions of bifurcation.

These results may lead, if combined with other information, such as suitable a priori bounds, to new multiplicity results. Yet, since the prescribed mean curvature operator is non-uniformly elliptic, it is not easy to establish such a priori bounds, even in specific examples; to obtain multiplicity results far from the local bifurcation solutions is therefore our future research goal.

The paper is organized as follows. In Section 2, we give the main theorems. Section 3 provides the proofs.

2. Main results

Notations. Denote by $\lambda_{1,\Omega}$ and $\varphi_{1,\Omega}$ the principal eigenvalue and eigenfunction of $-\Delta$ with the Dirichlet boundary condition on the domain Ω. We will write λ_1 and φ_1 for short, unless otherwise indicated. Denote by Λ the set of all simple eigenvalues of $-\Delta$ with the Dirichlet boundary condition on the domain Ω. Clearly, $\lambda_1 \in \Lambda \neq \varnothing$. Denote by λ_s and u_s the derivatives of $\lambda(s)$ and $u(s)$ with respect to s, respectively.

The main results of this paper are the following theorems.

Theorem 2.1. *Assume that $f \in C^k(-\delta, \delta)$ with $k \geqslant 1$, $f(0) = 0$ and $f'(0) > 0$ in (1.1). Let $\lambda_0 \in \Lambda$ and let φ_0 be the corresponding eigenfunction. Then $(\frac{\lambda_0}{f'(0)}, 0)$ is a bifurcation point and there exist an interval $(-\varepsilon, \varepsilon)$ and C^k functions $\lambda : (-\varepsilon, \varepsilon) \to \mathbb{R}^1$ and $\psi : (-\varepsilon, \varepsilon) \to Z$ such that*

$$-\,\mathrm{div}\Big(\frac{\nabla(s\varphi_0 + s\psi(s))}{\sqrt{1 + |\nabla(s\varphi_0 + s\psi(s))|^2}}\Big) = \lambda(s) f(s\varphi_0 + s\psi(s)), \tag{2.1}$$