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Desensitizing controls for semilinear wave equations☆

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ABSTRACT

The desensitizing control problems for semilinear wave equations are studied. When the nonlinearity f satisfies $\lim_{|s|\to\infty} \frac{f(s)}{|s|\ln^2|s|} = \lim_{|s|\to\infty} \frac{f'(s)}{\ln^2|s|} = 0$, we obtain the existence of a control which desensitizes the boundary observation. The proof is based on the combination of fixed-point arguments and explicit observability estimates for the linearized wave equation with a potential that depends on both x and t. The work solves the problem proposed by Tebou (2011) in the one-dimensional setting. For the interior observation, we assume that the nonlinearity is globally Lipschitz, under which the existence of ε-desensitizing controls is proved. Our proof is based on the fixed-point arguments and the unique continuation property.

1. Introduction

Let $T > 0, \Omega = (0,1), Q = \Omega \times (0,T)$ and $\Sigma = \{0,1\} \times (0,T)$. Let ω and $\mathcal{O}$ be two open subsets of Ω and 1_ω stands for the characteristic function of ω. We assume that $\omega \cap \mathcal{O} \neq \emptyset$ throughout this paper. For any $y^0 \in H_0^1(\Omega), y^1 \in L^2(\Omega)$ and $\xi \in L^2(Q)$, consider the following semilinear wave equation:

$$\begin{cases} y_{tt} - y_{xx} + f(y) = h1_\omega + \xi \quad \text{in } Q, \\ y = 0 \quad \text{on } \Sigma, \\ y(0) = y^0 + \tau_0 \hat{y}^0, \quad y_t(0) = y^1 + \tau_1 \hat{y}^1 \quad \text{in } \Omega, \end{cases} \tag{1.1}$$

where ω is the control region, $h \in L^2\big(\omega \times (0,T)\big)$ is the locally distributed control, $(\hat{y}^0, \hat{y}^1) \in H_0^1(\Omega) \times L^2(\Omega)$ with

$$\|\hat{y}^0\|_{H_0^1(\Omega)} = \|\hat{y}^1\|_{L^2(\Omega)} = 1 \tag{1.2}$$

are unknown, and $\tau_0, \tau_1 \in \mathbb{R}$ are small and unknown. ξ can be viewed as a given force source acting on the body.

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Let Φ and Ψ be the functionals defined on the solution y of the system (1.1):

$$\Phi(y) = \frac{1}{2}\int_0^T y_x^2(t,1)\mathrm{d}t, \tag{1.3}$$

$$\Psi(y) = \frac{1}{2}\int_0^T \int_{\mathcal{O}} y^2 dxdt. \tag{1.4}$$

One can understand Φ and Ψ to be observations of the solutions made at the right end $x = 1$ and in the set $\mathcal{O}$ during a time interval $[0, T]$, respectively.

Given a control h we say that the control h desensitizes Φ if for every pair $(\hat{y}^0, \hat{y}^1)$ satisfying (1.2) one has

$$\frac{\partial \Phi(y)}{\partial \tau_0}\Big|_{\tau_0=\tau_1=0} = \frac{\partial \Phi(y)}{\partial \tau_1}\Big|_{\tau_0=\tau_1=0} = 0; \tag{1.5}$$

given $\varepsilon > 0$, the control h is said to be ε-desensitize ψ if for every pair $(\hat{y}^0, \hat{y}^1)$ satisfying (1.2) one has

$$\left|\frac{\partial \Psi(y)}{\partial \tau_0}\Big|_{\tau_0=\tau_1=0}\right| \le \varepsilon \quad \text{and} \quad \left|\frac{\partial \Psi(y)}{\partial \tau_1}\Big|_{\tau_0=\tau_1=0}\right| \le \varepsilon. \tag{1.6}$$

We also say the system (1.1) is desensitizing controllable (resp., ε-desensitizing controllable or approximately desensitizing controllable). When (1.5) (resp., (1.6)) holds the functional Φ (resp., Ψ) is insensitive (resp., approximately insensitive) to the perturbations $\tau_0\hat{y}^0$ and $\tau_1\hat{y}^1$. So roughly speaking, the insensitivity problem (resp. ε-insensitivity problem) means that we are expected to find a local force source h such that the observation is almost invariant (resp., of very small change) with respect to small perturbations on the initial data.

The concept of ε-desensitizing control was proposed by Bodart and Fabre in [2] as a weakened notion of desensitizing control introduced by J. L. Lions in [11]. Desensitizing or ε-desensitizing controllability have been studied extensively for parabolic equations (see [2,3,17,12,7,13,14]) subsequent to [11] and [2]. However, up to now there are few papers to concern the desensitizing or ε-desensitizing controllability for wave equations. To our best knowledge, there are only four papers [16,15,4,1] studying the problem of desensitizing controls for wave equations. The paper [16] addresses the problem of desensitizing controls of a wave equation where the nonlinearity may grow at infinity in a superlinear way. The author of [16] proved a suitable observability inequality for a coupled system of wave equations, which assured desensitizing controllability. The author of [4] studied the boundary desensitizing controllability and interior ε-desensitizing controllability for one-dimensional linear wave equation observed in some open set. Since the method developed in [4] critically relied on the time periodicity of the one-dimensional linear wave equation, it does not apply to the general cases, for instance, the wave with a potential depending on both x and t.

In this paper we aim to find a control h that desensitizes (resp., ε-desensitizes) the observations Φ (resp., Ψ). For the former, due to the nonlinear character of the problem, new difficulty arises, and the method of [16,15,4] does not apply. We use another method partially from [21] and [8] to have the first theorem. For the latter, a convex duality theory (see [6]) helps us to have the ε-desensitizing controllability as stated in Theorem 1.2.

Theorem 1.1. *Let $0 \le \beta < 1$. Set $\omega = (\beta, 1)$. Assume that $f \in C^1(\mathbb{R})$ satisfies $f(0) = 0$ and*

$$\lim_{|s|\to\infty} \frac{f(s)}{|s|\ln^2|s|} = \lim_{|s|\to\infty} \frac{f'(s)}{\ln^2|s|} = 0. \tag{1.7}$$

Let $T > 2\beta$. Then for any $y^0 \in H_0^1(\Omega)$ and $y^1 \in L^2(\Omega)$, there exists a control $h \in L^2(Q)$ for (1.1) with $\xi = 0$ that desensitizes the functional Φ.