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Zero surface tension limit of the free-surface incompressible Euler equations with damping

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ABSTRACT

We consider the free boundary problem for a layer of an incompressible inviscid fluid in a uniform gravitational field, lying above a rigid bottom and below the atmosphere in a horizontally periodic setting. We establish the global-in-time zero surface tension limit of the problem with damping for the small initial data.

1. Introduction

1.1. Eulerian formulation

We consider an incompressible inviscid fluid evolving in a moving domain

$$\Omega(t) = \left\{ y \in \mathbb{T}^2 \times \mathbb{R} \mid -b < y_3 < h(t, y_1, y_2) \right\}. \tag{1.1}$$

We assume that the domain is horizontally periodic for $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ the usual 1-torus. The lower boundary of $\Omega(t)$ is assumed to be rigid and given by the constant $b > 0$, but the upper boundary is a free surface that is the graph of the unknown function $h : \mathbb{R}_+ \times \mathbb{T}^2 \to \mathbb{R}$. For each $t > 0$, the fluid is described by its velocity and pressure functions, which are given by $u(t, \cdot) : \Omega(t) \to \mathbb{R}^3$ and $p(t, \cdot) : \Omega(t) \to \mathbb{R}$, respectively. For each

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$t > 0$, we require that (u, p, h) satisfy the free-surface incompressible Euler equations with damping

$$\begin{cases} \partial_t u + u \cdot \nabla u + \nabla p + au = -ge_3 & \text{in } \Omega(t) \\ \operatorname{div} u = 0 & \text{in } \Omega(t) \\ \partial_t h = u_3 - u_1 \partial_1 h - u_2 \partial_2 h & \text{on } \{y_3 = h(t, y_1, y_2)\} \\ p = p_{atm} - \sigma H & \text{on } \{y_3 = h(t, y_1, y_2)\} \\ u_3 = 0 & \text{on } \{y_3 = -b\}. \end{cases} \tag{1.2}$$

The third equation in (1.2) states that the free surface moves with the velocity of the fluid. In Eqs. (1.2), $a > 0$ is the damping coefficient, $g > 0$ is the strength of gravity, p_{atm} is the constant pressure of the atmosphere and $\sigma > 0$ is the surface tension coefficient. Finally, H is twice the mean curvature of the free surface given by the formula

$$H = \nabla \cdot \left(\frac{\nabla h}{\sqrt{1 + |\nabla h|^2}} \right). \tag{1.3}$$

To complete the statement of the problem, we must specify the initial conditions. We suppose that the initial upper boundary is given by the graph of the function $h(0) = h_0 : \mathbb{T}^2 \to \mathbb{R}$, which yields the initial domain $\Omega(0)$ on which the initial velocity $u(0) = u_0 : \Omega(0) \to \mathbb{R}^3$ is specified. We will assume that $h_0 > -b$ on $\mathbb{T}^2$.

1.2. Background

The early works for the free-surface incompressible Euler equations (without damping, *i.e.*, $a = 0$) were focused on the irrotational fluids (*i.e.*, $\operatorname{curl} u = 0$), which began with Nalimov [20] of the local well-posedness for the small initial data and was generalized to the general initial data by Wu [25,26]. For the full free-surface incompressible Euler equations, the first local well-posedness was obtained by Lindblad [18] for the case without surface tension and by Coutand and Shkoller [7] for the case with (and without) surface tension, see also Christodoulou and Lindblad [6], Schweizer [21], Shatah and Zeng [22], Zhang and Zhang [29] and Coutand and Shkoller [8]. We also refer to Masmoudi and Rousset [19] and Wang and Xin [24] for the local well-posedness as the inviscid limit of the free-surface incompressible Navier–Stokes equations. We may refer to Wu [27,28], Germain, Masmoudi and Shatah [9,10], Ionescu and Pusateri [14,15] and Alazard and Delort [1] for the global well-posedness of the irrotational flows for the small initial data. With damping, the author [16,17] proved the global well-posedness of the general flows for the small initial data for the cases without and with surface tension, respectively.

For the zero surface tension limit of the free-surface incompressible Euler equations, we refer to Ambrose and Masmoudi [3,4] for the irrotational flows, and Shatah and Zeng [22] and Wang and Xin [24] for the general flows. Note that the results in [3,4,22,24] are all local in time. In this paper, with damping and gravity, we will prove the global-in-time zero surface tension limit of the free-surface incompressible Euler equations for the small initial data. We mention that, motivated by the global well-posedness in Beale [5] and Guo and Tice [11–13], the global-in-time zero surface tension limit of the free-surface incompressible Navier–Stokes equations has been established by Tan and Wang [23]; the results highly rely on the regularizing effect of the viscosity.

1.3. Reformulation in flattening coordinates

In order to transform the free boundary problem (1.2) to one in the fixed domain, we will use a flattening transformation. To this end, we define the fixed domain

$$\Omega := \mathbb{T}^2 \times (-b, 0), \tag{1.4}$$