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Global solutions of 3D axisymmetric Boussinesq equations with nonzero swirl

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ABSTRACT

Considering the 3D axisymmetric Boussinesq system with nonzero swirl, we obtain the global existence and uniqueness of the strong solutions (u, ρ), when $\|r^d u_0^\theta\|_{L^{\frac{3}{1-d}}}$, $d \in [0, 1]$, is sufficiently small. Furthermore, if $u_0 \in L^{\frac{3}{2}}(\mathbb{R}^3)$ and $ru_0^\theta \in L^1(\mathbb{R}^3) \cap L^2(\mathbb{R}^3)$, we have the decay estimate

$$\|u(t)\|^2_{L^2(\mathbb{R}^3)} + \langle t\rangle^2 \|\rho(t)\|^2_{L^2(\mathbb{R}^3)} + \langle t\rangle^2 \|u^\theta(t)\|^2_{L^2(\mathbb{R}^3)} \le C\langle t\rangle^{-\frac{1}{2}},$$

for any $t > 0$. At last, we get several continuation criteria.

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1. Introduction

In this paper, we consider the following 3D Boussinesq equations,

$$\begin{cases} \partial_t u + u \cdot \nabla u - \nu \Delta u + \nabla p = \rho e_3, \quad (t, x) \in \mathbb{R}^+ \times \mathbb{R}^3, \\ \partial_t \rho + u \cdot \nabla \rho - \kappa \Delta \rho = 0, \\ \nabla \cdot u = 0, \\ (u, \rho)|_{t=0} = (u_0, \rho_0), \end{cases} \tag{1.1}$$

where u, p, ρ represent velocity, pressure, temperature respectively, κ, ν are nonnegative dissipation parameters and e_3 is the vertical unit vector of $\mathbb{R}^3$. The Boussinesq equations, which model geophysical flows such as atmospheric fronts and oceanic circulation, play an important role in the study of Raleigh–Bernard convection (see [7,23,29,32] etc. for more details).

Considering the 2D standard Boussinesq equations, one can obtain the global well-posedness result similar to the 2D incompressible Navier–Stokes equations [24]. But the global regularity result is open to the 2D inviscid Boussinesq equations expect ρ is zero. Recently, mathematicians began to study the 2D Boussinesq system with fractional dissipations, please see [4,11,13–15,18,25] and the references therein. For the 3D

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Boussinesq system, we can only obtain the local well-posedness result or the global regularity result with respect to small initial data, see [8–10,19,33] etc.

As for the axisymmetric 3D Boussinesq system, we assume (u,ρ) are the axisymmetric solutions to the system (1.1), satisfying

$$u(t,x)=u^r(t,r,x_3)e_r+u^\theta(t,r,x_3)e_\theta+u^3(t,r,x_3)e_3 \quad \text{and} \quad \rho(t,x)=\rho(t,r,x_3),$$

where

$$e_r=(\frac{x_1}{r},\frac{x_2}{r},0),\quad e_\theta=(-\frac{x_2}{r},\frac{x_1}{r},0),\quad e_3=(0,0,1),\quad r=\sqrt{x_1^2+x_2^2}.$$

For simplicity, we assume $\nu=1$ in system (1.1). Then, we can equivalently reformulate the system (1.1) as

$$\begin{cases}\dfrac{\widetilde{D}}{Dt}u^r-(\partial_r^2+\partial_3^2+\dfrac{1}{r}\partial_r-\dfrac{1}{r^2})u^r-\dfrac{(u^\theta)^2}{r}+\partial_r p=0,\\ \dfrac{\widetilde{D}}{Dt}u^\theta-(\partial_r^2+\partial_3^2+\dfrac{1}{r}\partial_r-\dfrac{1}{r^2})u^\theta+\dfrac{u^\theta u^r}{r}=0,\\ \dfrac{\widetilde{D}}{Dt}u^3-(\partial_r^2+\partial_3^2+\dfrac{1}{r}\partial_r)u^3+\partial_3 p=\rho,\\ \partial_t\rho+u\cdot\nabla\rho-\kappa\Delta\rho=0,\\ \partial_r u^r+\dfrac{u^r}{r}+\partial_3 u^3=0,\\ (u^r,u^\theta,u^3,\rho)|_{t=0}=(u_0^r,u_0^\theta,u_0^3,\rho_0),\\ \dfrac{\widetilde{D}}{Dt}=\partial_t+u^r\partial_r+u^3\partial_3.\end{cases}\tag{1.2}$$

One particular case is the axisymmetric velocity without swirl, namely $u^\theta\equiv 0$. T. Hmidi and F. Rousset [16] obtained a unique global solution (u,ρ) to the system (1.2) with $u^\theta=0$ satisfying

$$\begin{cases}u\in C(\mathbb{R}^+;H^1)\cap L^2_{loc}(\mathbb{R}^+;H^2)\cap L^1_{loc}(\mathbb{R}^+;B^1_{\infty,1}),\\ \dfrac{\omega}{r}\in L^\infty_{loc}(\mathbb{R}^+;L^2),\quad \rho\in C(\mathbb{R}^+;L^2\cap B^0_{3,1}) \text{ and } \omega=\nabla\times u,\end{cases}$$

when the initial data satisfy

$$\begin{cases}u_0\in H^1 \text{ is an axisymmetric vector without swirl and } \nabla\cdot u_0=0,\\ \dfrac{\omega_0}{r}\in L^2,\quad \rho_0\in L^2\cap B^0_{3,1} \text{ is an axisymmetric function and } \omega_0=\nabla\times u_0.\end{cases}$$

Later, they [17] also obtained a unique global solution (u,ρ) to the system (1.2) with $u^\theta=0$ satisfying

$$\begin{cases}(u,\rho)\in C(\mathbb{R}^+;H^s)\times C(\mathbb{R}^+;H^{s-2}\cap L^m)\cap L^1_{loc}(\mathbb{R}^+;W^{1,\infty}),\\ r^2\rho\in C(\mathbb{R}^+;L^2),\end{cases}$$

when the initial data satisfy

$$\begin{cases}u_0\in H^s\ (s>\dfrac{5}{2}) \text{ is an axisymmetric vector without swirl and } \nabla\cdot u_0=0,\\ \rho_0\in H^{s-2}\cap L^m\ (m>6) \text{ is an axisymmetric function and } r^2\rho_0\in L^2.\end{cases}$$

Recently, there are some interesting works about the global well-posedness for the axisymmetric Boussinesq system with horizontal dissipation, see [26,27]. For the 3D axisymmetric Navier–Stokes equations with $u^\theta\neq 0$, there are many important works, refer to [6,12,20,22,28,31,34] for more details. In this paper, we will study the 3D axisymmetric Boussinesq system with $u^\theta\neq 0$ and also give the decay estimates for the global solutions (u,ρ).

At first, we list the local well-posedness result and energy estimates to the system (1.1) without proof here. Indeed, using the Friedrichs method and classical energy method similar to the methods from [2], we can obtain the following theorem.

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