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Monotone solutions of a nonlinear differential equation for geophysical fluid flows

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We prove the existence of monotone bounded solutions of a recently derived nonlinear differential equation of geophysical relevance. Our approach relies on functional-analytic techniques and the obtained results have wide applicability, which we illustrate by some examples.

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1. Introduction

The mathematical study of geophysical flows is currently of great interest since an in-depth understanding of the ongoing dynamics is essential in predicting features of these large-scale natural phenomena. While most studies of geophysical fluid flows ignore the Earth's shape as an oblate sphere and work with local approximations in planar flat-geometry, the quest to obtain accurate models leads to the consideration of the effects of spherical geometry. For flows in polar regions this type of consideration is particularly important. Arctic gyres are large systems of ocean currents caused by the Coriolis effect due to the Earth's rotation—see the discussion in [16,17]. While gyres are also encountered at mid-latitudes, they do not occur at the Equator [1], see the discussions in [6] and [7]. Recently, an approach that models ocean gyres as shallow water flows on a rotating sphere was presented in [10]. Using the stereographic projection, this model for arctic gyres in spherical coordinates can be transformed into a planar elliptic boundary-value problem that we investigate in the present paper using a functional-analytic approach.

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2. Preliminaries

In a gyre the typical ratio of vertical speed to horizontal speed is about 10^{-4} so that, neglecting vertical motion, one can introduce a stream function and model gyres as shallow water flows on a rotating sphere (see [10]). Using the stereographic projection, the model in spherical coordinates can be transformed into a planar elliptic partial differential equation with Dirichlet boundary conditions (see [10]). There are no gyres near the Equator, where strong currents with flow-reversal prevail (see the discussion in [6,9]) and we do not investigate gyres at mid-latitudes, where the specific geographic location and the corresponding topography of the ocean basin play an important role in the flow dynamics. In this paper we are concerned with arctic gyres centered at the North Pole, for which it is reasonable to neglect azimuthal variations in the horizontal flow velocity, so that the elliptic boundary-problem can be transformed into a second-order differential equation on a semi-infinite interval, with suitable asymptotic conditions (see [2]). Note that the situation in the southern polar region is quite different since the terrestrial South Pole is surrounded by a large land mass that is encircled by a strong current with vertical flow variation (see [8] and [18]). Moreover, since the Arctic Polar region is covered by ice, waves play no role, whereas in the Southern Ocean the interaction of currents and waves is significant (see the discussion in the book [23] and the recent paper [11]). The linear setting for arctic gyres which present only a dependence on the polar angle was recently analyzed in detail in [2], where also some explicit solutions were obtained. In [4], the existence, uniqueness and the stability of solutions to a Lipschitz model for the flow vorticity were investigated by studying the equivalent integral equation on a semi-infinite interval. Using fixed point techniques, the existence and uniqueness of bounded solutions to such an integral equation were studied in [3]. In this paper we are interested in some qualitative aspects of the flow in an arctic gyre, more specifically, in the question of the existence of stagnant flow regions. In this paper, we establish the existence of strictly monotone bounded solutions for a given continuous vorticity, using a fixed point approach. The results are relevant for the behavior of the ocean flows in arctic gyres. Note that the study of the existence of (monotone) bounded solutions for differential equation has a long history—we refer the reader to [5,14,20–22,24,25] and the references therein. However, the problem that arises in the context of arctic gyres presents some specific challenges and opportunities.

Let us first briefly describe the model for gyres [10]. In spherical coordinates, with $\theta \in [0, \pi)$ the polar angle (with $\theta - \pi/2$ the conventional angle of latitude, so that $\theta = 0$ corresponds to the South Pole) and $\varphi \in [0, 2\pi)$ the angle of longitude (or azimuthal angle), the horizontal gyre flow on the spherical Earth has azimuthal and polar velocity components given by

$$\frac{1}{\sin\theta}\,\psi_\phi \quad \text{and} \quad -\,\psi_\theta\,, \tag{1}$$

respectively, where $\psi(\theta, \varphi)$ is the stream function. Setting

$$\psi(\theta,\varphi) = -\omega\,\cos\theta + \Psi(\theta,\varphi),$$

where Ψ is the stream function associated with the vorticity of the motion of the ocean that is not driven by the Earth's rotation, the governing equation for the gyre flow is

$$\frac{1}{\sin^2\theta}\,\Psi_{\varphi\varphi} + \Psi_\theta \cot\theta + \Psi_{\theta\theta} = F(\Psi - \omega\,\cos\theta).$$

Using the stereographic projection from the South Pole to the plane of the Equator, given by

$$\xi = r\,e^{i\,\phi} \quad \text{with} \quad r = \cot\Big(\frac{\theta}{2}\Big) = \frac{\sin\theta}{1-\cos\theta}, \tag{2}$$

where $\theta \in (0, \pi]$ is the polar angle (with $\theta = \pi$, i.e. with $r = 0$, representing the North Pole), and $\phi \in [0, 2\pi)$ is the azimuthal angle in the Earth's spherical coordinates, while (r, ϕ) are the polar coordinates in the plane

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