Nonlinear Analysis 165 (2017) 198–242

Contents lists available at ScienceDirect

Nonlinear Analysis

www.elsevier.com/locate/na

Strong solutions of Stochastic models for viscoelastic flows of Oldroyd type

Utpal Manna*, Debopriya Mukherjee

School of Mathematics, Indian Institute of Science Education and Research (IISER) Thiruvananthapuram, Thiruvananthapuram 695016, Kerala, India

ARTICLE INFO

Article history:
Received 22 May 2017
Accepted 5 October 2017
Communicated by S. Carl

MSC:
60H15
60H30
76A05
76A10
76D03

Keywords:
Oldroyd fluid
Maximal strong solution
Lévy noise
Commutator estimates

ABSTRACT

In this work we study stochastic Oldroyd type models for viscoelastic fluids in $\mathbb{R}^d, d = 2, 3$. We show existence and uniqueness of strong local maximal solutions when the initial data are in H^s for $s > d/2, d = 2, 3$. Probabilistic estimate of the random time interval for the existence of a local solution is expressed in terms of expected values of the initial data.

1. Introduction

Over the past few years, there have been many works devoted to viscoelastic fluids in dimensions two and three. Most of these works are concerned about local existence of strong solutions, global existence of weak solutions, necessary condition for blow-up (in the spirit of well-known Beale–Kato–Majda criterion [4]) and global well-posedness for smooth solutions with small initial data.

In this work, we focus upon the classical Oldroyd type models for viscoelastic fluids (see Oldroyd [50]) in $\mathbb{R}^d, d = 2, 3$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} - \nu \Delta \mathbf{v} + \nabla p = \mu_1 \nabla \cdot \tau \quad \text{in} \quad \mathbb{R}^d \times (0, T), \tag{1.1}$$

$$\frac{\partial \tau}{\partial t} + (\mathbf{v} \cdot \nabla)\tau + a\tau + \mathbf{Q}(\tau, \nabla \mathbf{v}) = \mu_2 \mathcal{D}(\mathbf{v}) \quad \text{in} \quad \mathbb{R}^d \times (0, T), \tag{1.2}$$

* Corresponding author.
E-mail addresses: manna.utpal@iisertvm.ac.in (U. Manna), debopriya13@iisertvm.ac.in (D. Mukherjee).

https://doi.org/10.1016/j.na.2017.10.003
0362-546X/

$$\nabla \cdot \mathbf{v} = 0 \quad \text{in} \quad \mathbb{R}^d \times (0, T), \tag{1.3}$$

$$\mathbf{v}(0, \cdot) = \mathbf{v}_0, \ \tau(0, \cdot) = \tau_0 \quad \text{in} \quad \mathbb{R}^d. \tag{1.4}$$

Here $\mathbf{v}$ is the velocity vector field which is assumed to be divergence free, τ is the non-Newtonian part of the stress tensor (i.e., $\tau(x,t)$ is a (d,d) symmetric matrix), p is the pressure of the fluid, which is a scalar. The parameters ν (the viscosity of the fluid), a (the reciprocal of the relaxation time), μ_1 and μ_2 (determined by the dynamical viscosity of the fluid, the retardation time and a) are assumed to be non-negative. $\mathcal{D}(\mathbf{v})$ is called the deformation tensor and is the symmetric part of the velocity gradient

$$\mathcal{D}(\mathbf{v}) = \frac{1}{2}(\nabla \mathbf{v} + \nabla^t \mathbf{v}).$$

$\mathbf{Q}$ is a quadratic form in $(\tau, \nabla \mathbf{v})$. As remarked in Chemin and Masmoudi [14], since the equation for the stress tensor should be invariant under coordinate transformation, $\mathbf{Q}$ cannot be most general quadratic form, and for Oldroyd fluids one usually chooses

$$\mathbf{Q}(\tau, \nabla \mathbf{v}) = \tau \mathcal{W}(\mathbf{v}) - \mathcal{W}(\mathbf{v})\tau - b\left(\mathcal{D}(\mathbf{v})\tau + \tau \mathcal{D}(\mathbf{v})\right),$$

where $b \in [-1, 1]$ is a constant and $\mathcal{W}(\mathbf{v}) = \frac{1}{2}(\nabla \mathbf{v} - \nabla^t \mathbf{v})$ is the vorticity tensor, and is the skew-symmetric part of velocity gradient.

There is growing literature devoted to these systems and it is almost impossible to provide a complete review on the topic. We shall restrict ourselves to a few significant works which are relevant to our paper.

1.1. Deterministic Oldroyd models

In comparison to the classical evolution equations, analysis of the above model is significantly difficult due to the lack of diffusion in the τ Eq. (1.6) and structure of $\mathbf{Q}$. To be a little more precise, one of the key difficulties in proving local existence with diffusion only in the $\mathbf{v}$ equation stems from the nonlinear terms. Since H^s is an algebra for $s > d/2$, so one obtains

$$|\langle (\mathbf{v} \cdot \nabla)\tau, \varphi \rangle_{H^s}| \le \|\mathbf{v}\|_{H^s} \|\nabla \tau\|_{H^s} \|\varphi\|_{H^s}.$$

Thus we must estimate $\|\nabla \tau\|_{H^s}$, and if we start with $\tau_0 \in H^s$ we do not have any control over the H^s norm of $\nabla \tau$ due to lack of H^{s+1} bound for τ. Due to the same reasons, the semigroup method to mild solutions may not work in this case and also the local m-accretivity property is not available due to the absence of a diffusive term. However, similar issues appear in Euler equation or in semi-dissipative/ideal magnetohydrodynamic systems and there are sufficient literature (see e.g. Fefferman et al. [17]) suggesting how to tackle this issue. On the other hand, due to the special structure of $\mathbf{Q}$, the formal L^2-energy estimate of the system (1.1)–(1.4) appears as following:

$$\frac{1}{2}\frac{d}{dt}(\mu_2 \|\mathbf{v}(t)\|_{\mathrm{L}^2}^2 + \mu_1 \|\tau(t)\|_{\mathrm{L}^2}^2) + \nu \mu_2 \|\nabla \mathbf{v}(t)\|_{\mathrm{L}^2}^2 + a\mu_1 \|\tau(t)\|_{\mathrm{L}^2}^2 \le |b| \ \|\nabla \mathbf{v}(t)\|_{\mathrm{L}^\infty} \|\tau(t)\|_{\mathrm{L}^2}^2.$$

Since by the Brezis–Wainger type logarithmic Sobolev inequality, L^∞-norm of gradient of velocity field can be bounded by that of vorticity field for the Sobolev exponent strictly bigger than $d/2 + 1$, the difficulty here arises in getting an L^∞ estimate on the vorticity. Indeed, at first glance it seems to be hopeless because the vorticity equation involves a transport term as well as a nonlocal term. However, one needs to perform a losing estimate (see, Chemin–Masmoudi [14]) for the transport equation satisfied by τ that allow us to obtain a Beale–Kato–Majda (see [4]) type sufficient condition of non-breakdown.

Due to the parabolic–hyperbolic coupling and the special structure of $\mathbf{Q}$, the corresponding stationary problem is also interesting and was studied by Renardy [55]. The existence and uniqueness of local strong