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Existence and stability of stationary solution to compressible Navier–Stokes–Poisson equations in half line

Lei Wang, Guojing Zhang, Kaijun Zhang*

School of Mathematics and Statistics, Northeast Normal University, Changchun 130024, PR China

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ABSTRACT

In this paper, we investigate the asymptotic stability of the stationary solution to the outflow problem for the compressible Navier–Stokes–Poisson system in a half line. We show the existence of the stationary solution with the aid of the stable manifold theory. The time asymptotic stability of the stationary solution is obtained by the elementary energy method. Furthermore, for the supersonic flow at spatial infinity, we also obtain an algebraic and an exponential decay rate, when the initial perturbation belongs to the corresponding weighted Sobolev space. The proof is based on a time and space weighted energy method.

1. Introduction

The compressible Navier–Stokes–Poisson (called NSP in the sequel for simplicity) system may be used to simulate the transport of charged particles under the influence of electrostatic force governed by the self-consistent Poisson equation (cf. [2]). In this paper, we consider the following NSP equations in the half line $\mathbb{R}_+ := (0, \infty)$:

$$\begin{cases} \rho_t + (\rho u)_x = 0, & \text{(a)} \\ (\rho u)_t + (\rho u^2 + p(\rho))_x = \mu u_{xx} + \rho E, & \text{(b)} \\ E_x = \rho - \rho_+, & \text{(c)} \end{cases} \tag{1.1}$$

* Corresponding author.
E-mail addresses: wanglei345@126.com (L. Wang), zhangkj201@nenu.edu.cn (K. Zhang).

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together with the initial conditions

$$\begin{aligned} &(\rho, u)(0, x) = (\rho_0, u_0)(x), \qquad \lim_{x\to\infty} (\rho_0, u_0)(x) = (\rho_+, u_+), \\ &\inf_{x\in\mathbb{R}_+} \rho_0(x) > 0,\ \ \rho_+ > 0, \end{aligned} \tag{1.2}$$

and the boundary conditions

$$\begin{cases} u(t,0) = u_b < 0, & \text{(a)} \\ E(t,\infty) = 0. & \text{(b)} \end{cases} \tag{1.3}$$

Here the unknown functions are the density ρ, the velocity u and the electric field E. The pressure p is assumed to be a function depending only on the density given by

$$p = p(\rho) = K\rho^\gamma, \tag{1.4}$$

for $K > 0$ and $\gamma \geq 1$. In Eqs. (1.1), the positive constants μ and ρ_+ represent the viscosity coefficient and the doping profile, respectively. In addition, u_+ and u_b are also constants. Further we suppose that the compatibility conditions on $u(t, x)$ of order 0 and 1 hold at the origin $(0, 0)$. It is worth noticing that from the electric field equation (1.1)(c) and the boundary data (1.3)(b), the initial value $E_0 := E(0, x)$ can be determined by the initial density ρ_0. The condition (1.3)(a) implies that the fluid blows out through the boundary $\{x = 0\}$. This is the reason why the problem (1.1)–(1.3) is called the outflow problem in some literature. In this case, the characteristic speed of the hyperbolic equation (1.1)(a) for the density ρ is negative around the boundary $\{x = 0\}$, so we do not need the assumption of $\rho(t, 0) = \rho_b$ for the well-posedness.

Recently, some important progress was made for the compressible NSP system. The global existence and the optimal time convergence rates of the classical solution around a constant state were obtained in [8,13,7,25]. The global strong solution to the one-dimensional non-isentropic NSP system with large data for density-dependent viscosity was established by Tan–Yang–Zhao–Zou [21]. The pointwise estimate of the solution was discussed in [23]. The global well-posedness in the Besov type space for the NSP system was also proved in [6]. From the above work, it is not difficult to find the elementary fact that the momentum of the NSP system decays at the slower rate than that of the compressible Navier–Stokes system in the absence of the electric field. This fully demonstrates that the electric field could affect the large time behavior of the solution. In addition, the quasi-neutral limit and some related asymptotic limits were considered in [4,22], and the global existence and nonexistence were discussed in [3,1].

In general, the large-time behavior of solutions to (1.1) in the half space is much more complicated than that for the Cauchy problem, cf. [9,12,14–16] and references therein. For the single quasineutral Navier–Stokes system (1.1)(a) and (1.1)(b) with $E = 0$, Kawashima, Nishibata and Zhu [11] proved the existence and the asymptotic stability of the stationary solution for the outflow problem. The convergence rate for this stability result was obtained by Nakamura, Nishibata and Yuge in [19], if the initial perturbation decays in a spatial direction. Furthermore, when the doping profile is zero, Duan–Yang [5] proved the stability of rarefaction wave and boundary layer for the outflow problem on the two-fluid NSP equations. Zhou–Li [26] studied the convergence rate of corresponding solutions toward the same boundary layer. As a continuation of the above study, the generalization of this one-dimensional outflow problem to the multi-dimensional half space problem was studied by Kagei, Kawashima, Nakamura and Nishibata in [18,10].

The main concern of the present paper is to extend these results to the compressible NSP system with the nonzero doping profile. Specifically, we find a new stationary solution called the boundary layer and show the asymptotic stability of the stationary solution as well as the convergence rate for the outflow problem (1.1)–(1.3). We focus on the influence of the electric field on the existence and the stability of the stationary solution. Compared to the isentropic model in [11], our problem is more general and more complex for the electric field is taken into account. For instance, in order to obtain the existence of stationary