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Optical soliton solutions to Fokas-lenells equation using some different methods

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A B S T R A C T

Dark and bright with singular solitons shall be yielded to Fokas–Lenells equation which describes soliton dynamics in optical fibers. The two integration schemes that are applied in this context are the modified simple equation method and the trial equation method. Additional solutions, besides, optical solitons, are recovered.

1. Introduction

Optical solitons form the backdrop of research in the area of telecommunications industry. These soliton molecules form the information carrier across inter-continental distances over the globe. There are several models that describe this phenomena mathematically, for instance, nonlinear Schrödinger's equation (NLSE) while others are Manakov model, Chen–Lee–Liu equation, Gerdjikov–Ivanov model, Lakshmanan–Porsezian–Daniel equation, Radhakrishnan–Kundu-Lakshmanan model, Sasa–Satsuma model, complex Ginzburg–Landau equation, Schrödinger–Hirota equation and many more. We shall address a different model which has gained popularity for the past few years since its first appearance about a decade ago. This is the Fokas–Lenells equation (FLE) [1–10]. This is an alternate model to study soliton dynamics through a polarization–preserving optical fiber and is well-known as a generalized type of NLSE with cubic law nonlinearity. The trial and the modified simple equation approaches are two integration techniques that will study this model. We shall yield some special type optical soliton solutions via these schemes to FLE along with their respective existence criteria.

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1.1. Governing model

The dimensionless form of FLE throughout perturbation terms is given by [1–10]

$$\mathrm{i}q_t + a_1 q_{\mathrm{xx}} + a_2 q_{\mathrm{xt}} + |q|^2(\mathrm{b}q + i\sigma q_x) = i\,[\alpha q_x + \lambda(|q|^{2m}q)_x + \mu(|q|^{2m})_x q]. \tag{1}$$

In (1), the first term is the temporal evolution of the pulses and $q(x, t)$ is the complex-valued soliton profile. The coefficient of a_1 is group velocity dispersion (GVD) and the coefficient of a_2 stands for spatio-temporal dispersion (STD). The nonlinear term is given by the coefficient of b, which represents Kerr law nonlinearity. α corresponds to the inter-modal dispersion, λ comes from the self-steepening effect when μ is nonlinear dispersion. Lastly, m comes from the full nonlinearity exponent.

2. Iteration of trial equation method

The fundamental stages of this method are enumerated by following steps [11,12]:

Step-1: Let's consider a nonlinear evolution equation (NLEE)

$$P(q, q_t, q_x, q_{\mathrm{tt}}, q_{\mathrm{xt}}, q_{\mathrm{xx}}, \ldots) = 0. \tag{2}$$

Eq. (2) transforms to ordinary differential equation (ODE)

$$\Delta(Q, Q', Q'', Q''', \ldots) = 0 \tag{3}$$

by help of the wave variable $q(x, t) = Q(\zeta)$, $\zeta = x - vt$, where $Q = Q(\zeta)$ is an dependent function, Δ is a polynomial in Q and its subsequent derivatives.

Step-2: In what follows, we consider the following auxiliary first order ODE

$$(Q')^2 = H(Q) = \sum_{j=0}^{M} \delta_i Q^i \tag{4}$$

where δ_i, $(i = 0, 1, \ldots, M)$ are unknown coefficients and will be later fixed. Plugging Eq. (4) and necessary derivatives terms into Eq. (3) gives a polynomial expression in $\Phi(Q)$. Employing the balancing rule, the value of M is located. Setting the coefficients of $\Phi(Q)$ to zero, one comes up with a overdeterminet system which contains unknown parameters. With help of symbolic computation softwares, the constants of v, δ_0, δ_1, … and δ_M are fixed.

Step-3: Reformulate Eq. (4) with an integral representation as

$$\pm(\zeta - \zeta_0) = \int \frac{\mathrm{d}Q}{\sqrt{H(Q)}} \tag{5}$$

Based on the classification of the discriminants of the integrand, one categorizes the roots of $H(Q)$, and evaluate the integral Eq. (5). So, analytical solutions for Eq. (2) are recovered.

2.1. Implementation to the perturbed FLE

In order to construct the solutions of Eq. (1) by the trial equation method, the solution form will be assumed as

$$q(x, t) = Q(\zeta)e^{i\phi(x,t)}, \tag{6}$$

where the transformation ζ is given as follows

$$\zeta = x - \mathrm{vt}. \tag{7}$$

Here, $Q(\xi)$ is the soliton amplitude component and v is the soliton speed, while the soliton phase component $\phi(x, t)$ is presented as follows

$$\phi(x, t) = -\kappa x + \omega t + \theta, \tag{8}$$

where θ is the soliton phase constant, κ is the soliton frequency when ω is the soliton wave number.

Plugging (6) into (1), the real and imaginary components give rise to

$$(a_1 - a_2 v)Q'' - (\omega + \kappa^2 a_1 - \kappa\omega a_2 + \alpha\kappa)Q + (\kappa\sigma + b)Q^3 - \kappa\lambda Q^{2m+1} = 0, \tag{9}$$

$$(\kappa \mathrm{v} a_2 + \omega a_2 - v - 2\kappa a_1 - \alpha)Q' + \sigma Q'Q^2 - ((2m + 1)\lambda + 2m\mu)Q'Q^{2m} = 0. \tag{10}$$

respectively. From the imaginary component, one can conclude the restriction

$$m = 1 \tag{11}$$

and

$$\sigma = (2m + 1)\lambda + 2m\mu \tag{12}$$

while the soliton speed is