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Optical solitons and other solutions to the conformable space–time fractional Fokas–Lenells equation

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ABSTRACT

This study reveals the dark, bright, combined dark–bright, singular optical solitons and other solutions to the conformable space–time fractional Fokas–Lenells equation. We use two integral schemes in reaching such solutions. The constraints conditions which guarantee the existence of valid solutions are stated. By choosing the suitable values of the parameters involved, we plot the 2-dimensional and 3-dimensional surfaces to some of the obtained solutions.

1. Introduction

Soliton is known to describe the particle-like properties of pulses propagating in a nonlinear medium, they are present in different kind of nonlinear partial differential equations that describe several complex phenomena in nonlinear science, such as fluid dynamics, nonlinear optics, condensed matter, acoustics, plasma physics, convictive fluids, solid-state physics and so on [1,2]. Nonlinear Schrödinger equations are prototypical dispersive nonlinear partial differential equations that have been derived in many areas of physics and analyzed mathematically for over three decades [3]. These type of equation play a vital in scientific fields, it is therefore very to seek for their wave solutions. Several computational approaches for constructing wave solutions to this kind off equations have been submitted to the literature, such as the novel (G'/G)-expansion method [4], the extended Fan sub-equation method [5], the trial equation method [6], the extended simple equation method [7], the modified simple equation approach [8], the improved Bernoulli sub-equation function method [9], the Lie group analysis [10], Riccati–Bernoulli's sub-ODE method [11], extended Jacobi's elliptic function approach [12], the improved $\tan(\phi(\eta)/2)$-expansion method [13] and many others [14–65].

However, this research is aimed at constructing various fractional optical solitons to the conformable space–time fractional Fokas–Lenells equation by using two integral schemes, namely; the extended sinh-Gordon equation expansion method (ShGEEM) [66–68] and the modified $\exp(-\Psi(\eta))$-expansion function method (MEFM) [69–72].

The conformable fractional Fokas–Lenells equation is given as follows

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$$\begin{aligned}&\mathrm{i}D_t^{\alpha}\Phi + a_1 D_x^{2\beta}\Phi + a_2 D_t^{\alpha} D_x^{\beta}\Phi + |\Phi|^2 (b\Phi + i\sigma D_x^{\beta}\Phi) - i\delta D_x^{\beta}\\&- i\rho D_x^{\beta}(|\Phi|^{2n}\Phi) - i\gamma\Phi D_x^{\beta}(|\Phi|^{2n}) = 0, \quad 0 < \alpha, \quad \beta \le 1,\end{aligned} \tag{1.1}$$

where $i = \sqrt{-1}$, Φ is a complex-valued function of x; spatial variable and t; temporal. The first term in Eq. (1.1) represents the linear fractional temporal evolution of the pulses in nonlinear optical fibres. The coefficients a_1, a_2, δ, ρ and γ are the spatio-temporal dispersion (STD), group velocity dispersion (GVD), inter-modal dispersion (IMD), self-steepening perturbation term and nonlinear dispersion (ND) coefficient, respectively. The parameter n represents the full nonlinearity of Eq. (1.1) [73].

When $\alpha = \beta = 1$, Eq. (1.1) becomes the original Fokas–Lenells equation given in [73–76].

2. Conformable fractional derivative

For some past years, fractional differential equations became one of the important area of research due to their wider range of applications in the various fields of nonlinear science [77]. Various definitions of fractional derivatives have been submitted to literature, amongst are Riemann–Liouville, Caputo and Grunwald–Letnikov definitions, Atangana–Baleanu derivative in Caputo sense, Atangana–Baleanu fractional derivative in Riemann–Liouville sense [78,79] and the recently developed conformable fractional derivative [80].

Here, some basic definition, properties and theorem about conformable fractional derivative are discussed [80].

Definition 1. Let g: $(0, \infty) \to \mathbb{R}$, then the conformable fraction derivative of g of order α is defined as

$$T_{\alpha}(g)(t) = \lim_{\epsilon \to 0} \frac{g(t + \epsilon t^{1-\alpha}) - g(t)}{\epsilon}, \quad t > 0, \quad 0 < \alpha < 1. \tag{2.1}$$

1. $T_{\alpha}(bg + ch) = bT_{\alpha}(g) + cT_{\alpha}(h)$, $b, c \in \mathbb{R}$,
2. $T_{\alpha}(t^{\lambda}) = \lambda t^{\lambda-\alpha}$, $\lambda \in \mathbb{R}$,
3. $T_{\alpha}(gh) = gT_{\alpha}(h) + hT_{\alpha}(g)$,
4. $T_{\alpha}(\frac{g}{h}) = \frac{\mathrm{hT}_{\alpha}(g) - \mathrm{gT}_{\alpha}(h)}{h^2}$,
5. if g is differentiable, then $T_{\alpha}(g)(t) = t^{1-\alpha}\frac{\mathrm{dg}}{\mathrm{dt}}$.

Theorem 1. *Let* g, h: $(0, \infty) \to \mathbb{R}$ *be differentiable and also* α *differentiable functions, then the following rule holds:*

$$T_{\alpha}(g \circ h)(t) = t^{1-\alpha} h'(t) g'(h(t)). \tag{2.2}$$

3. Applications

This sections presents the applications of the two integral schemes in obtaining the fractional optical solitons and other solutions to Eq. (1.1).

Consider the complex fractional travelling wave transformation

$$\Phi(x, t) = \psi(\eta) e^{i\Theta}, \quad \eta = \nu\left(\frac{x^{\beta}}{\beta} - c\frac{t^{\alpha}}{\alpha}\right), \quad \Theta = -k\frac{x^{\beta}}{\beta} + \omega\frac{t^{\alpha}}{\alpha} + \theta. \tag{3.1}$$

Substituting Eq. (3.1) into Eq. (1.1), yields

$$\nu^2(a_1 - a_2 c)\psi'' + (a_2 k\omega - \omega - a_1 k^2 - \delta k)\psi + (b + k\sigma)\psi^3 - k\rho\psi^{1+2n} = 0 \tag{3.2}$$

from the real part and

$$(c + \delta + 2a_1 k - a_2(\mathrm{ck} + \omega) - \sigma\psi^2 + (\rho + 2n\rho + 2n\gamma)\psi^{2n})\psi' = 0 \tag{3.3}$$

from the imaginary part.

Considering $n = 1$ [73], Eqs. (1.1), (3.2) and (3.3) become

$$\begin{aligned}&\mathrm{i}D_t^{\alpha}\Phi + a_1 D_x^{2\beta}\Phi + a_2 D_t^{\alpha} D_x^{\beta}\Phi + |\Phi|^2 (b\Phi + i\sigma D_x^{\beta}\Phi) - i\delta D_x^{\beta}\\&- i\rho D_x^{\beta}(|\Phi|^2\Phi) - i\gamma\Phi D_x^{\beta}(|\Phi|^2) = 0, \quad 0 \le \alpha, \quad \beta \le 1,\end{aligned} \tag{3.4}$$

$$\nu^2(a_1 - a_2 c)\psi'' + (a_2 k\omega - \omega - a_1 k^2 - \delta k)\psi + (b - k(\rho - \sigma))\psi^3 = 0 \tag{3.5}$$

and

$$(c + \delta + 2a_1 k - a_2(\mathrm{ck} + \omega) + (3\rho + 2\gamma - \sigma)\psi^2)\psi' = 0, \tag{3.6}$$

respectively.

Setting $(3\rho + 2\gamma - \sigma) = 0$ in Eq. (3.6), we get the following relations:

$$\sigma = 3\rho + 2\gamma \tag{3.7}$$

and