Optik - International Journal for Light and Electron Optics 172 (2018) 79–85

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Optical soliton perturbation with Kundu–Eckhaus equation by exp $(-\phi(\xi))$-expansion scheme and G'/G^2-expansion method

Saima Arshed[a], Anjan Biswas[b,c], Mahmoud Abdelaty[d], Qin Zhou[e,*], Seithuti P. Moshokoa[c], Milivoj Belic[f]

[a] Department of Mathematics, University of the Punjab, Lahore 54590, Pakistan
[b] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Deanship of Research and Graduate Studies, Applied Science University, PO Box 5055, Bahrain
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[f] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Dark solitons
Singular solitons
integrability

ABSTRACT

This paper reveals dark and singular optical solitons, as well as their combinations thereof, of the perturbed Kundu–Eckhaus equation. Two integration methodologies are adopted here. The existence criteria for these solitons are also presented.

1. Introduction

Optical soliton perturbation is the backbone of telecommunications industry. This industry stays in business because of the marvel of soliton transmission technology. One of the various models that govern these pulse transmission across inter–continental distances is the Kundu–Eckhaus (KE) equation. In the past, several approaches were applied to address this model [1–10]. This paper studies the model with a few Hamiltonian perturbation terms by using two integration schemes. These are $\exp(-\phi(\xi))$-expansion scheme and G'/G^2-expansion method. The dark and singular soliton solutions will emerge from these integration schemes. These soliton solutions will be listed with their corresponding constraint conditions that will formulate the existence criteria of these pulses. The paper starts with an introduction to the model followed by a quick review of the two algorithms. Finally, the soliton solutions to the model will be computed. The details are discussed in the rest of the paper.

1.1. Governing equation

Consider the perturbed KE equation given as

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.ijleo.2018.07.019
Received 8 May 2018; Accepted 4 July 2018

$$iu_\tau + au_{xx} + b|u|^4u = i[\alpha u_x + \lambda_1(|u|^2u)_x + \mu_1(|u|^2)_xu]. \tag{1}$$

In Eq. (1), x and τ represent spatial and temporal co–ordinates respectively. The dependent variable is $u(x, \tau)$ which gives the pulse profile. The first term in (1) is accounts for temporal evolution of the pulse, while the real-valued constants a, and b represents group velocity dispersion, quintic nonlinearity. On the right-hand side of (1), α denotes the inter-modal dispersion, λ_1 gives the effect of self–steepening for short pulses to eliminate the shock formation and μ_1 is the higher–order dispersion coefficient.

2. A quick glance at proposed analytical techniques

A general form of nonlinear evolution equation (NLEE) is:

$$P(u, D_\tau u, D_x u, D_\tau^2 u, D_{x\tau} u, D_x^2 u, \ldots) = 0, \tag{2}$$

where $u = u(x, \tau)$ is an unknown function, P is a polynomial in u as well as its partial derivatives. Here, nonlinear terms and its highest order derivatives are included. The traveling wave hypothesis is the transformation

$$u(x, \tau) = U(\xi), \quad \xi = x - v\tau.$$

After applying this wave transformation, the NLEE converts to a nonlinear ordinary differential equation (ODE) as given by:

$$S(U, U', U'', U''', \ldots) = 0, \tag{3}$$

where $'$ denotes the derivative with respect to ξ.

2.1. exp($-\phi(\xi)$)-expansion method

For exp($-\phi(\xi)$)-expansion method, the wave forms are expressed as

$$U(\xi) = \sum_{n=0}^{N} F_n(\exp(-\Phi(\xi)))^n, \tag{4}$$

where F_n are unknown constants to be determined and $\Phi(\xi)$ satisfies the auxiliary ODE:

$$\Phi'(\xi) = \exp(-\Phi(\xi)) + \mu\exp(\Phi(\xi)) + \lambda. \tag{5}$$

The auxiliary Eq. (5) has the general solutions given by one of the following five forms:

Case-1:(Hyperbolic function solutions)
When $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, then

$$\Phi_1(\xi) = \ln\left[\frac{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + C)\right) - \lambda}{2\mu}\right]. \tag{6}$$

Case-2: (Trigonometric function solutions)
If $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, then

$$\Phi_2(\xi) = \ln\left[\frac{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + C)\right) - \lambda}{2\mu}\right]. \tag{7}$$

Case-3: (Hyperbolic function solutions)
However, if $\lambda^2 - 4\mu > 0$ and $\mu = 0$ and $\lambda \neq 0$ then

$$\Phi_3(\xi) = -\ln\left[\frac{\lambda}{\cosh(\lambda(\xi + C)) + \sinh(\lambda(\xi + C)) - 1}\right]. \tag{8}$$

Case-4: (Rational function solutions)
Next, if $\lambda^2 - 4\mu = 0$ and $\mu \neq 0$ and $\lambda \neq 0$,

$$\Phi_4(\xi) = \ln\left[-\frac{2(\lambda(\xi + C)) + 2}{\lambda^2(\xi + C)}\right]. \tag{9}$$

Case-5:
Finally, if $\lambda^2 - 4\mu = 0$ and $\mu = 0$ and $\lambda = 0$, then

$$\Phi_5(\xi) = \ln(\xi + C). \tag{10}$$

where C is the integration constant.