Optik - International Journal for Light and Electron Optics 172 (2018) 144–149

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Chirped singular and combo optical solitons for Gerdjikov–Ivanov equation using three integration forms

Anwar Ja'afar Mohamad Jawad[a], Anjan Biswas[b,c], Mahmoud Abdelaty[d], Qin Zhou[e,*], Seithuti P. Moshokoa[c], Milivoj Belic[f]

[a] Al-Rafidain University College, Baghdad 10014, Iraq
[b] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Deanship of Research and Graduate Studies, Applied Science University, PO Box 5055, Bahrain
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[f] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370*Keywords:*
Dark solitons
Singular solitons
Integrability

ABSTRACT

This study carries out the integration of Gerdjikov–Ivanov equation. The csch method, the extended tanh–coth method, and the modified simple equation method are utilized to extract the analytical soliton solutions.

1. Introduction

The dynamics of the propagation of soliton molecules through a variety of waveguides, such as optical couplers, fibers, PCF and metamaterials, form the backbone of telecommunication industry. These soliton molecules have been studied with a variety of models that stem from the fundamental equation in electromagnetics, namely the Maxwell's equations. The most visible model is the well known nonlinear Schrödinger's equation (NLSE). There are several other models that are studied in this context. These emerge from the fundamental NLSE and are typically referred to as derivative NLSE (DNLSE). Three forms of DNLSE are known as of today. This paper will address one such DNLSE that is the third form of DNLSE or DNLSE-III, alternatively known as the Gerdjikov–Ivanov (GI) equation. There has been extensive research conducted in this equation for the past couple of decades [1–20]. Three integration schemes will be applied to study GI equation from the integration perspective. These algorithms will reveal soliton solutions which is an important commodity for soliton industry. After a quick introduction to the model, the integration schemes will be implemented and the extracted soliton solutions will be presented and classified.

1.1. Governing model

The dimensionless form of GI equation that is studied in this paper takes the form [1]:

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.ijleo.2018.07.013
Received 7 May 2018; Accepted 4 July 2018

$$\mathrm{i}q_t + \mathrm{a}q_{xx} + b|q|^4 q + \mathrm{i}\,\mathrm{c}q^2 q_x^* = 0 \tag{1}$$

where, in (1), $q(x, t)$ represents complex-valued wave function with x and t being independent spatial and temporal variables respectively. The first term is linear temporal evolution, while a is the coefficient of the group velocity dispersion (GVD) and b gives quintic nonlinearity effect. The coefficient of c is a form of nonlinear dispersion. The parameters a, b and c are all real-valued constants.

1.2. Traveling waves

Traveling waves are waves of permanent form. Here, in (1), the traveling wave hypothesis is:

$$q(x, t) = e^{i\theta(x,t)}u(\xi) \tag{2}$$

where $\xi = x - \gamma t$ and the phase portion is $\theta(x, t) = -kx + \omega t + \varphi(\xi)$ and $u(\xi)$ is the amplitude component of the wave. Here, γ is its speed, k is the soliton frequency and ω is its wave number. Next, we have defining the following derivatives:

$$\begin{aligned} q_t &= e^{i\theta}[-\gamma u' + i(\omega - \gamma\,\varphi')u] \\ q_x &= e^{i\theta}[u' - i(k - \varphi')u] \\ q_{xx} &= e^{i\theta}[u'' - i(ku' - u\varphi'' - u'\varphi') - i(k - \varphi')u' - (k - \varphi'^2)u] \end{aligned} \tag{3}$$

Eq. (1) can be decomposed into real and imaginary parts and that yields a pair of relations. The real part equation is

$$au'' - (\omega - \gamma\varphi' + ak^2 - 2\mathrm{a}\,\mathrm{k}\varphi' + a\varphi'^2)u + cu^3(k - \varphi') + bu^5 = 0 \tag{4}$$

and the imaginary part is

$$\mathrm{a}\,\mathrm{u}\varphi'' - [2\mathrm{ak} - 2a\varphi' - cu^2]u' = 0 \tag{5}$$

To solve the above equations, we choose the ansatz:

$$\varphi' = a_1 u^2 + b_1 \tag{6}$$

where a_1 and b_1 denote the constant and nonlinear chirp parameters, respectively. Substitute (6) in (5) to give

$$(4a_1 a + c)u^2 + (2\mathrm{ab}_1 - 2\mathrm{ak} - \gamma) = 0 \tag{7}$$

Then

$$a_1 = \frac{-c}{4a}, \quad b_1 = \frac{\gamma}{2a} + k \tag{8}$$

Therefore

$$\varphi' = \left(k + \frac{\gamma}{2a} - \frac{c}{4a}u^2\right) \tag{9}$$

On substituting (9) in Eq. (4), one recovers

$$u'' + \frac{\gamma^2 + 4\mathrm{ak}\gamma - 4a\omega}{4a^2}u - \frac{c\gamma}{2a^2}u^3 + b\left(\frac{(3c^2 + 16\mathrm{ab})}{(16a^2)}\right)u^5 = 0 \tag{10}$$

assume

$$n_1 = \frac{\gamma^2 + 4\mathrm{ak}\gamma - 4a\omega}{4a^2},\ n_2 = -\frac{c\gamma}{2a^2},\ n_3 = b\left(\frac{(3c^2 + 16\mathrm{ab})}{(16a^2)}\right) \tag{11}$$

Then Eq. (10) reduces to:

$$u'' + n_1 u + n_2 u^3 + n_3 u^5 = 0 \tag{12}$$

Now, set

$$V = u^2 \tag{13}$$

and then Eq. (12) transform to

$$2VV' - V'^2 + 4n_1 V^2 + 4n_2 V^3 + 4n_3 V^4 = 0 \tag{14}$$

This Eq. (14) will be analyzed in details in the next section using three different integration scehemes.

2. Application to GI equation

In this section Eq. (14) will be studied using csch method, extended tanh-coth method and the modified simple equation method (MSEM). These are detailed in the subsequent sub–sections.