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Homogenization of non-integer dimensional multilayered sphere

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ABSTRACT

Behavior of normalized polarizability of a multilayer sphere of non-integer dimensional dielectric materials placed in free space is presented. Normalized polarizability is calculated with respect to number of layers and value of non-integer dimensional parameter. Special case by taking value of non-integer dimensional parameter equal to three is obtained to validate present work by comparing it with already published work.

1. Introduction

Multilayer structures are commonly found in nature and also can be prepared artificially for a variety of applications. One interesting natural occurrence of these structures is the wings and protective coverings of insects. As the angle of incidence of light changes, different colors are observed giving the illusion of change in appearance. Initially artificial multilayer structures were prepared using glass slabs/lenses to produce interference of light at particular wavelength. Later, the study of behavior of light through these structures became basis for the field of optronics.

A variety of multilayer structures composed of different materials and shapes had been studied. Scattering form two concentric spheres using Mie theory was presented by Aden and Kerker [1]. Stable and accurate recursive equations to compute scattering coefficient for a multilayer sphere was presented by Wu and Wang [2]. Their work had limitations over number and thickness of layers and the problem was rectified by Kia and Massoli [3]. An extension to [2] was presented by Johnson to calculate internal field and absorption cross section by individual layers [4]. The mixing formula for scatterers of multilayer spherical structure was described in [5,6] using the Maxwell Garnett formula. Same procedure was extended for chiral sphere [7]. A dielectric sphere with arbitrary number of layers and fixed outermost radius was realized as a radial uni-axial sphere [8]. Furthermore, the effect of variation of effective permittivity of outermost layer and plasmon resonances was noted in realized uni-axial sphere. The case of plasmon resonance being effected by spherical geometry for two layer sphere was described in [9]. Quasi-static analysis has been used to solve electromagnetic scattering problem with assumption that the size of the object being very small as compared to the frequency of incident light [10,11]. The concept of non-integer dimensional space has been the subject of interest for long time as it helps in replacing complex structures by simple structures having non-integer dimensions. Structural axioms for non-integer dimensional space were presented in [12]. A detailed discussion on Guass's law in non-integer dimensional space is presented by Muslih and Baleanu [13]. Classical electromagnetic equations and physics laws give rise to new concepts with benefit of memory storage when solved in non-integer dimensional space [14]. Laplace's equation is solved and behavior of a dielectric sphere in non-integer dimensional space is given in [15]. Exact solution of the Laplace's equation for NID space in cylindrical coordinate system has been derived in [16]. The behavior of electromagnetic fields and waves in fractional dimensional space is thoroughly discussed in [17]. The scattering from plasmonic sphere placed in fractional dimensional space was analyzed by Noor et al. [18]. The concept of basic circuit elements NID space [19], behavior of radially anisotropic sphere in NID space [20], and cloaking and magnification of

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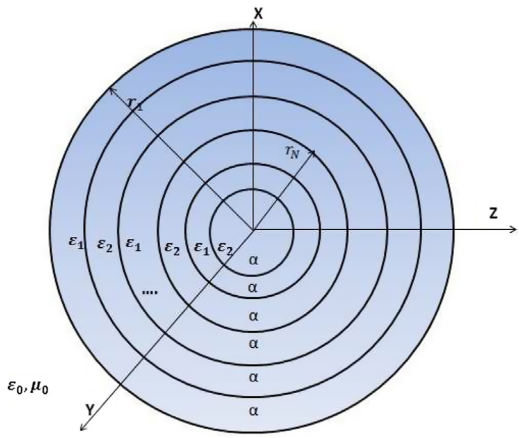

Fig. 1. Multilayer NID sphere in a region bounded by free space.

anisotropic cylinder and sphere in NID space are discussed by Nisar and Naqvi [21]. Quasi-static analysis of multilayered sphere in non-integer dimensional sphere placed in the free space in presented in this paper. Behavior of disturbances due to multilayer sphere is given with respect to non integer dimension parameter alpha of the sphere.

2. Mathematical formulation

Consider a multilayer sphere as shown in Fig. 1. It consists of N concentric layers of homogeneous dielectric medium each with uniform thickness. The outermost layer has radius r_1 and the radii, r_i, of the internal layers can be calculated using the following relation

$$r_i = \frac{N-(i-1)}{N} r_1, \qquad i = 1, 2, 3, \ldots, N \tag{1}$$

The multilayers have interchanging relative permittivities ϵ_1 and ϵ_2. It has been assumed that the medium inside the sphere is NID in radial direction of the spherical coordinate system. Non-integer dimension of the space has been represented by parameter α, where $2 \leq \alpha \leq 3$. The medium hosting the multilayer sphere is free space described by constitutive parameters ϵ_0 and μ_0. Under quasi-static assumption, it is assumed that the sphere is much smaller than the wavelength of the incident field. The multilayer NID sphere is excited by a uniform z-directed plane wave. The incident electric field under quasi-static limit, $k = \omega\sqrt{\mu_0 \epsilon_0} \to 0$, is

$$\mathbf{E}_{\text{inc}} = \hat{z} E_0 \tag{2}$$

Corresponding potential ϕ_{inc} can be derived from $\mathbf{E}_{inc} = -\nabla \phi_{nc}$ and is written below

$$\phi_{\text{inc}}(r, \theta) = -E_0 r \cos\theta \tag{3}$$

where (r, θ) are polar coordinates. Total potential outside the sphere is written below

$$\phi_0(r, \theta) = -E_0 r \cos\theta + D r^{-2} \cos\theta \tag{4}$$

First term is due to the incident potential and second term represents the disturbances in the free space due to the presence of multilayer NID sphere. D is the unknown co-efficient.

Electric potential $\phi_i(r, \theta)$ for ith layer of the multilayer NID sphere can be obtained by solving the Laplace's equation $\nabla_\alpha^2 \phi(r, \theta) = 0$ in the spherical coordinates. Using the first order approximation of [13], general expression for potential in the ith layer is assumed as

$$\phi_i(r, \theta) = -C_i r \cos\theta (\alpha - 2) + D_i r^{-(\alpha-1)} (\alpha - 2) \cos\theta \tag{5}$$

where C_i and D_i are unknown coefficients to be determined. These unknowns are determined by using the following boundary conditions.

$$\phi_i = \phi_{i+1}, \qquad r_i = r_{i+1} \tag{6}$$

$$\epsilon_i \frac{\partial \phi_i}{\partial r} = \epsilon_{i+1} \frac{\partial \phi_{i+1}}{\partial r}, \qquad r_i = r_{i+1} \tag{7}$$

The resulting algebraic equations can be arranged in matrix form according to transmission line method.

$$\begin{bmatrix} C_i \\ D_i \end{bmatrix} = \frac{1}{3\epsilon_i} M_i(\alpha) \begin{bmatrix} C_{i+1} \\ D_{i+1} \end{bmatrix} \tag{8}$$