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Optical soliton perturbation with Fokas-Lenells model by Riccati equation approach

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ABSTRACT

This paper employs Riccati equation method to obtain singular and combo optical soliton solutions to the Fokas-Lenells equation. The existence criteria of these solitons are also given. These soliton solutions to the model of study are being reported for the first time in this paper.

1. Introduction

The study of optical soliton perturbation in fibers, PCF, metamaterials and metasurfaces as well as optical couplers has been going on for decades and is still burning bright. This paper shines light on the perturbation of optical solitons that are modeled by a newly proposed model. This is the Fokas-Lenells equation (FLE) [1–20]. This model has been around for about a decade since it was first proposed [11]. A wide variety of integration techniques is visible and successfully applied to address this model for the past few years. These include modified simple equation method [3], trial equation method [3], extended trial function method [2], Kudryashov's method [5], tanh-coth method [10], rogue wave solutions [9], complex amplitude hypothesis solution [14]. This paper will address the perturbed FLE where the perturbation terms are all of Hamiltonian type and appear with full nonlinearity. The Riccati equation approach reveals chirp-free optical solitons that are of singular type as well as a mix-type that are commonly referred to as combo-optical solitons. The results of this paper will reveal novelty of the solutions.

1.1. Governing model

The dimensionless form of perturbed FLE that has been proposed takes the form [1,2]:

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$$\mathrm{i}q_t + a_1 q_{xx} + a_2 q_{xt} + |q|^2(\mathrm{B}q + i\sigma q_x) = i[\alpha q_x + \lambda(|q|^{2m}q)_x + \mu(|q|^{2m})_x q], \tag{1}$$

where the right hand side represent all of the Hamiltonian perturbation terms. In Eq. (1), the independent variables are represented with x and t that stand for spatial and temporal variables respectively and the dependent variable $q(x, t)$ governs the complex-valued soliton profile. Here, a_1 is the coefficient of group velocity dispersion while a_2 is the coefficient of spatio-temporal dispersion whose inclusion was recommended a few years ago in order to make the governing equation well-posed. Then, σ is the coefficient of nonlinear dispersion and B indicates the effect of self-phase modulation. On the right hand side, α gives inter-modal dispersion, while λ accounts for self-steepening effect to avoid the formation of shocks, and finally μ gives another type of nonlinear dispersion. The parameter m accounts for full nonlinearity.

2. Traveling wave hypothesis

The solutions of (1) may be assumed to be:

$$q(x, t) = e^{i\theta(x,t)}u(\eta), \tag{2}$$

where $\eta = x - \gamma t$ and the phase $\theta(x, t) = -kx + \omega t + \theta_0$, $u(\eta)$ is the amplitude component of the wave and γ is its speed. k is the soliton frequency, ω is its wave-number and θ_0 is the phase constant. Eq. (1) can be decomposing into real and imaginary parts yields a pair of relations. The real and imaginary parts of Eq. (1) are respectively

$$(a_1 - a_2)u'' + (a_2\omega k - a_1 k^2 - \alpha k - \omega)u + (B + k\sigma)u^3 - \mathrm{k}\,\mathrm{u}[(2m + 1)\lambda + 2m\mu]u^{2m} = 0, \tag{3}$$

and

$$\gamma + 2ka_1 - a_2(\gamma k + \omega) - \sigma u^2 + \alpha + [(2m + 1)\lambda + 2m\mu]u^{2m} = 0. \tag{4}$$

From (4)

$$[\lambda(2m + 1) + 2m\mu]u^{2m} = -\gamma - 2\mathrm{ka}_1 + a_2(\gamma k + \omega) + \sigma u^2 - \alpha. \tag{5}$$

For $(2m + 1)\lambda + 2m\mu = 0$, $\sigma = 0$, then

$$\lambda = \frac{-2m\mu}{(2m + 1)}, \quad \gamma = \frac{2\mathrm{ka}_1 - a_2\omega + \alpha}{(a_2 k - 1)}. \tag{6}$$

If $m = 1$, then

$$\lambda = \frac{-2\mu}{3}, \quad \gamma = \frac{2\mathrm{ka}_1 - a_2\omega + \alpha}{a_2 k - 1}. \tag{7}$$

For $m = 2$,

$$\lambda = \frac{-4\mu}{5}, \quad \gamma = \frac{2\mathrm{ka}_1 - a_2\omega + \alpha}{a_2 k - 1}. \tag{8}$$

Substitute (6) in (3) to give

$$(a_1 - a_2\gamma)u'' + (a_1 k^2 - \omega + k\gamma(1 - a_2 k))u + Bu^3 = 0. \tag{9}$$

For

$$n_1 = (a_1 - a_2\gamma), \quad n_2 = (a_1 k^2 - \omega + k\gamma(1 - a_2 k)), \tag{10}$$

hence (9) yields

$$n_1 u'' + n_2 u + Bu^3 = 0. \tag{11}$$

3. Application to FLE

Assume that the solution of (11) is in the form

$$u(\eta) = a_0 + \sum_{j=1}^{n} a_j\phi^j(\eta) + \sum_{j=1}^{n} b_j\phi^{-j}(\eta), \tag{12}$$

where $\phi(\eta)$ satisfies the Riccati equation:

$$\phi'(\eta) = P\phi^2(\eta) + Q\phi(\eta) + R, \tag{13}$$

and n is determined by balancing u'' and u^3, this gives $n = 1$. Accordingly, Eq. (13) becomes

$$u = a_0 + a_1\phi(\eta) + b_1\phi^{-1}(\eta). \tag{14}$$

Substituting (14) into (11), we obtain the following system of algebraic equations