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# Optical solitons with polarization mode dispersion for Lakshmanan–Porsezian–Daniel model by the method of undetermined coefficients

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ABSTRACT

This paper obtains bright, dark and singular optical soliton solutions to the Lakshmanan–Porsezian–Daniel model that describes soliton propagation through polarization-mode dispersive fibers, without the effect of four-wave mixing. The method of undetermined coefficients is employed to retrieve these soliton solutions. The existence criteria for these solitons are also presented.

## 1. Introduction

Optical solitons in polarization mode dispersion (PMD) fibers have been extensively studied with coupled nonlinear Schrödinger's equation and other similar models such as Manakov equations. This paper will be addressing the study of soliton solutions for PMD fibers with the vector coupled version of Lakshmanan–Porsezian–Daniel (LPD) model. LPD model has been extensively studied in polarization preserving fibers during the past few years [1–15] although lately this model has sparked interest to model soliton propagation in PMD fibers [2]. The coupled form of LPD is first derived without the effect of four-wave mixing (4WM) so that the coupled version of LPD stays simple. Subsequently, the method of undetermined coefficients is employed to retrieve bright, dark and singular soliton solutions to the model. These solitons appear with several forms of parametric restrictions that are written as constraint conditions. The details are enumerated in the subsequent sections.

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### 1.1. Governing model

The dimensionless form of LPD model, with Kerr law nonlinearity, that has been studied in the past is given in the form [1–9,15]:

$$iu_t + au_{xx} + bu_{xt} + c\,|u|^2u = \sigma u_{xxxx} + \alpha(u_x)^2u^* + \beta\,|u_x|^2u + \gamma\,|u|^2u_{xx} + \lambda u^2u^*_{xx} + \delta\,|u|^4u \tag{1}$$

In (1), the real-valued coefficients $a$ and $b$ represent group velocity dispersion (GVD) and spatio-temporal dispersion (STD) respectively. Then, $c$ is the coefficient of Kerr law nonlinearity and $\sigma$ is the fourth order dispersion while $\delta$ accounts for two-photon absorption. The remaining terms are from other forms of dispersive phenomenon [1]. Soliton formation is the outcome of a delicate balance between dispersive and nonlinear effects.

For birefringent fibers, the model therefore splits into two components leading to the coupled vector form of LPD. After neglecting the effects of 4WM, this coupled system takes the form [2]:

$$\begin{aligned} & iq_t + a_1q_{xx} + b_1q_{xt} + (c_1\,|q|^2 + d_1\,|r|^2)q \\ = \;& \sigma_1q_{xxxx} + (\alpha_1q_x^2 + \beta_1r_x^2)q^* + (\gamma_1\,|q_x|^2 + \delta_1\,|r_x|^2)q \\ + \;& (\lambda_1\,|q|^2 + \mu_1\,|r|^2)q_{xx} + (\xi_1q^2 + \eta_1r^2)q^*_{xx} + (f_1\,|q|^4 + g_1\,|q|^2|r|^2 + h_1\,|r|^4)q \end{aligned} \tag{2}$$

$$\begin{aligned} & ir_t + a_2r_{xx} + b_2r_{xt} + (c_2\,|r|^2 + d_2\,|q|^2)r \\ = \;& \sigma_2r_{xxxx} + (\alpha_2r_x^2 + \beta_2q_x^2)r^* + (\gamma_2\,|r_x|^2 + \delta_2\,|q_x|^2)r \\ + \;& (\lambda_2\,|r|^2 + \mu_2\,|q|^2)r_{xx} + (\xi_2r^2 + \eta_2q^2)r^*_{xx} + (f_2\,|r|^4 + g_2\,|r|^2|q|^2 + h_2\,|q|^4)r \end{aligned} \tag{3}$$

From (2) and (3), the coefficients of $a_j$ and $b_j$, for $j = 1, 2$ are GVD and STD respectively for the two components. The self-phase modulation (SPM) terms are $c_j$ and $f_j$ while the cross-phase modulation effect comes from the coefficients of $d_j$, $g_j$ and $h_j$. The fourth order dispersion stems from the coefficients of $\sigma_j$. The remaining terms give the effect of addition dispersion.

## 2. Mathematical analysis

With the system been established, we now proceed to analyzed it focusing in optical soliton solutions evolving inside birefringent fibers. In order to explore the integration of the LPD model for birefringent fibers governed by the system (2) and (3), a solution of the form

$$q(x, t) = P_1(x, t)e^{i\phi(x,t)} \tag{4}$$

$$r(x, t) = P_2(x, t)e^{i\phi(x,t)} \tag{5}$$

where

$$\phi(x, t) = -\kappa x + \omega t + \theta_0 \tag{6}$$

is assumed. Notably, phase-matching condition is implemented to permit integrability of the governing model. Here, $\kappa$ denotes the soliton frequency and $\omega$ is the soliton wave number, while the parameter $\theta_0$ indicates the phase constant. The substitution of (4) and (5) into the system (2) and (3), leads to an expression in which, after splitting into real and imaginary parts, the real portion takes the form:

$$\begin{aligned} & \sigma_l\frac{\partial^4P_l}{\partial x^4} - (a_l + 6\sigma_l\kappa^2)\frac{\partial^2P_l}{\partial x^2} - b_l\frac{\partial^2P_l}{\partial x\partial t} - (b_l\kappa\omega - \omega - a_l\kappa^2 - \sigma_l\kappa^4)P_l - [c_l + (\alpha_l + \lambda_l + \xi_l - \gamma_l)\kappa^2]P_l^3 \\ & + f_lP_l^5 + g_lP_l^3P_{\bar{l}}^2 + h_lP_lP_{\bar{l}}^4 - [d_l + (\beta_l + \eta_l + \mu_l - \delta_l)\kappa^2]P_lP_{\bar{l}}^2 + (\alpha_l + \gamma_l)P_l\left(\frac{\partial P_l}{\partial x}\right)^2 \\ & + (\beta_l + \delta_l)P_l\left(\frac{\partial P_{\bar{l}}}{\partial x}\right)^2 + (\lambda_l + \xi_l)P_l^2\frac{\partial^2P_l}{\partial x^2} + (\eta_l + \mu_l)P_{\bar{l}}^2\frac{\partial^2P_l}{\partial x^2} = 0 \end{aligned} \tag{7}$$

while the imaginary portion is

$$\begin{aligned} & (1 - b_l\kappa)\frac{\partial P_l}{\partial t} + (b_l\omega - 2a_l\kappa - 4\sigma_l\kappa^3)\frac{\partial P_l}{\partial x} - 2(\xi_l - \alpha_l - \lambda_l)\kappa P_l^2\frac{\partial P_l}{\partial x} \\ & - 2(\eta_l - \mu_l)\kappa P_{\bar{l}}^2\frac{\partial P_l}{\partial x} + 4\kappa\sigma_l\frac{\partial^3P_l}{\partial x^3} + 2\beta_l\kappa P_lP_{\bar{l}}\frac{\partial P_{\bar{l}}}{\partial x} = 0 \end{aligned}$$

where $\bar{l} = 3 - l$, for $l = 1, 2$. Its is well known that the soliton profile $P_l(x, t)$ can be written in the form $F(x - vt)$, where $v$ represents the soliton speed. In view of such fact, the imaginary part equation leads to

$$v = \frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa} \tag{8}$$

as long as the identities

$$\xi_l = \alpha_l + \lambda_l \tag{9}$$

$$\eta_l = \mu_l \tag{10}$$