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Singular and dark optical solitons to the well posed Lakshmanan–Porsezian–Daniel model

Ahmad Javid*, Nauman Raza

Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore, Pakistan

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ABSTRACT

This paper scrutinizes dark, singular and periodic singular solitons to the well posed Lakshmanan–Porsezian–Daniel model to study the soliton propagation in nonlinear optics. Three types of nonlinear media namely, Kerr, parabolic and anti cubic law nonlinearity are investigated. The versatile integration gadget, modified simple equation method, is employed in this work. A number of constraint relations emerge to guarantee the existence of these soliton solutions.

1. Introduction

The passage of optical solitons in various kinds of nonlinear waveguides has been an engaging issue in the nonlinear optics. Solitons show up in almost every aspect of our daily life. The study of optical solitons is crucial in nonlinear fibre optics [1–19]. Solitons also play a pivotal role in background of neuroscience [20,21]. The issue of soliton propagation is addressed by various kinds of nonlinear evolution equations such as nonlinear Schrödinger equation, Kundu–Eckhaus equation, Schrödinger–Hirota equation, Biswas–Milovic equation, Ginzburg–Landau equation, Manakov equation, Sasa–Satsuma equation and others [22–31]. Multiple results have been reported in the context of Lakshmanan–Porsezian–Daniel (LPD) model [32–51]. In this work, the dynamics of soliton propagation is surveyed with the advocacy of LPD model which was reported two decades ago. The inclusion of spatio-temporal dispersion (STD) in originally reported LPD model along with group velocity dispersion (GVD) makes the problem well posed. It was highlighted in 2012 [52,53].

The spotlight of this paper is the integrability aspect of this model with Kerr, parabolic and anti cubic laws of nonlinearity. The modified simple equation method fetches dark, singular and periodic singular solutions for this model. The constraint conditions are also surfaced during the process. The details are given in ensuing sections.

2. Overview of the method

Consider the general form of nonlinear partial differential equation (PDE)

$$Q(p, p_t, p_x, p_{tt}, p_{xt}, p_{xx}, \dots) = 0, \tag{1}$$

where Q is the polynomial in $p(x, t)$ and its corresponding partial derivatives as well as nonlinear terms. The process has four major stages:

(2) **Step 1:** Consider the transformation

* Corresponding author.
E-mail address: ahmadjavid001@gmail.com (A. Javid).

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$$p(x,t)=q(\xi),\quad \xi=k(x-\mathrm{vt}),\tag{2}$$

where v and k are the constants, to convert Eq. (5) in the subsequent ODE:

$$S(q,q',q'',q''',\ldots)=0.\tag{3}$$

Here S stands for the polynomial in $q(\xi)$ as well as its ordinary derivatives i.e. $'=\frac{d}{d\xi}$.

(3) **Step 2:** It is inferred that Eq. (3) accepts the subsequent form of the solution

$$q(\xi)=\sum_{i=0}^{N}a_i\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^i,\tag{4}$$

where the unknowns to be found out are a_i and $\psi(\xi)$, provided $a_N\neq 0$.

(4) **Step 3:** The homogeneous balance principle is the tool used to determine the unknown N incorporating the nonlinearities and highest order derivative in Eq. (3).

(5) **Step 4:** By plugging Eq. (4) into Eq. (3), a polynomial in $\frac{\psi'(\xi)}{\psi(\xi)}$ is gained in which the coefficients of $\psi^{-j}=0,1,2,\ldots$ are collected and are compared with zero to form an algebraic system of nonlinear equations. This system is solved for the unknowns a_i and $\psi(\xi)$ which finally retrieves the exact solution of Eq. (1).

3. The governing equation

The dimensionless form of the LPD model with higher and spatio-temporal dispersion is considered in this work [23].

$$\mathrm{i}p_t+\mathrm{a}p_{\mathrm{xx}}+\mathrm{b}p_{\mathrm{xt}}+\mathrm{c}F(|P|^2)p=\sigma p_{\mathrm{xxxx}}+\alpha(p_x)^2p^*+\beta|p_x|^2p+\gamma|p|^2p_{\mathrm{xx}}+\lambda p^2p^*_{\mathrm{xx}}+\delta|p|^4p.\tag{5}$$

The dependent variable $p(x,t)$ in above system stands for the complex valued wave function. The first term on the left side represents the temporal evolution of the nonlinear wave, while a and b are the coefficient of the group velocity dispersion and spatio-temporal dispersion respectively. The functional F, the source of nonlinearity, is a smooth real-valued function such that $F(|p^2|)p: C\mapsto C$. Treating C as a linear space R^2, the function $F(|p|^2)p$ is r-times continuously differentiable function such that

$$F(|p|^2)p\in\bigcup_{m,n=1}^{\infty}C^r((-n,n)\times(-m,m);R^2).\tag{6}$$

On the other side of equality, δ and σ represent the two-photon absorption and fourth order dispersion. In the upcoming section, we present the procedure to construct soliton solutions for the model given in Eq. (5) for different kinds of nonlinearities.

In order to accelerate the integrability scheme, we start by letting

$$p(x,t)=q(\xi)e^{i\phi},\quad \xi=k(x-\mathrm{vt}),\tag{7}$$

where $q(x,t)$ represents the frame of the wave profile and the phase is defined as

$$\phi(x,t)=-\kappa x+\omega t+\theta,\tag{8}$$

where κ, ω and θ represents the soliton frequency, wave number and phase constant respectively. So, by using Eq. (7) in Eq. (5) and splitting real

$$\sigma(k^4q^{(\mathrm{iv})}-6\kappa^2k^2q'')+q(\alpha+\beta)(k^2(q')^2-\kappa^2q^2)+q^2(\gamma+\lambda)(k^2q''\\ -\kappa^2q)+\delta q^5+(\omega+a\kappa^2-b\omega\kappa)q+(\mathrm{bk}^2v-\mathrm{ak}^2)q''-\mathrm{cF}(q^2)q=0,\tag{9}$$

and imaginary parts

$$(-1+b\kappa)\mathrm{vkp}'-(2a\kappa-\omega b+4\sigma\kappa^3)\mathrm{kq}'+4\sigma\kappa k^3q'''+2(\alpha+\gamma-\lambda)\kappa\mathrm{kq}^2q'=0.\tag{10}$$

Setting the coefficients of linearly independent functions to zero in Eq. (10), we have

$$\sigma=0.\tag{11}$$

$$\lambda=\alpha+\gamma.\tag{12}$$

Using above two results, we have from Eq. (10)

$$v=\frac{2a\kappa-\omega b}{b\kappa-1},\tag{13}$$

whenever

$$1\neq b\kappa,\tag{14}$$

which is the soliton velocity and is independent of the nonlinear medium in question. Similarly, using Eqs. (11) and (12) in Eq. (9), we have

$$k^2(\mathrm{bv}-a)q''+(a\kappa^2-(1-\mathrm{bk})\omega)q-(2\lambda+\beta)\kappa^2q^3+(\gamma+\lambda)k^2q^2q''+(\alpha+\beta)k^2(q')^2q-\mathrm{cF}(q^2)q+\delta q^5=0.\tag{15}$$

Eq. (15) will be used to construct soliton solutions for different forms of nonlinear mediums.