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Optical soliton perturbation with Radhakrishnan–Kundu–Lakshmanan equation by traveling wave hypothesis

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ABSTRACT

This paper obtains chirp-free bright optical soliton solutions to Radhakrishnan–Kundu–Lakshmanan equation by traveling wave hypothesis along with the technical criteria for the existence of such solitons. The speed of the soliton also emerges from the constraint condition.

1. Introduction

Optical soliton perturbation has been studied with several models that are known today. One such model that is considered in this context is the Radhakrishnan–Kundu–Lakshmanan (RKL) equation that has been around for quite a few years. There are several integration techniques that have been applied to handle this equation or other models to address soliton perturbation [1–10]. One of the most classic approaches is traveling wave hypothesis. In this assumption, a wave of permanent form is hypothesized and subsequently, the analysis is carried out after integrating the reduced version of the governing differential equation. This gives way to the extraction of the speed of the soliton along with the soliton solution. One major disadvantage of this tool is that the method fails to retrieve the soliton radiation component that is always possible to obtain from the other classic method, namely inverse scattering transform. But, as always, the simplicity of traveling wave hypothesis algorithm is always given a preferential treatment. The rest of the paper discusses the algorithm for RKL equation with Kerr and power law nonlinearity.

2. Governing model

This section will study the model in two sections that depends on the type of nonlinearity. The study will be split into two subsections.

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2.1. Kerr-law nonlinearity

The perturbed RKL in its dimensionless form is [1,3–10]:

$$iq_t + aq_{xx} + b\,|q|^2q = i[\alpha q_x + \lambda(|q|^2q)_x + \theta(|q|^2)_x q - \gamma q_{xxx}] \tag{1}$$

where the wave variable is represented by the complex valued dependent variable $q(x, t)$ with x and t being the independent variables that represents the spatial and temporal variables respectively. The first term represents linear evolution, while a is the coefficient of group velocity dispersion (GVD) and b represents the nonlinear term that is of Kerr type. On the right hand side, α is the inter-modal dispersion that is considered in addition to chromatic dispersion. The next term with λ accounts for self-steepening effect that is included to avoid the formation of shock waves while θ gives the effect of nonlinear dispersion. Finally, γ stands for third order dispersion (3OD) whenever GVD carries a low count. This paper will retrieve bright soliton solutions of the model by the aid of traveling wave hypothesis.

2.1.1. Mathematical analysis

To start off, the solution hypothesis to (1) is given by [1–5]:

$$q(x, t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where

$$s = x - vt, \tag{3}$$

and v is the speed of the wave. The phase component of the pulse is defined as:

$$\phi(x, t) = -\kappa x + \omega t + \theta_0, \tag{4}$$

where, κ is the frequency, ω is the wave number and θ_0 is the phase constant.

On substituting (2) into (1) leads to two components due to real and imaginary parts. The real part equation reads:

$$(a + +3\gamma\kappa)g'' - (\omega + \alpha\kappa + a\kappa^2 + \gamma\kappa^3)g + (b - \lambda\kappa)g^3 = 0 \tag{5}$$

where $g' = dg/ds$ and $g'' = d^2g/ds^2$. The imaginary part equation is:

$$3\gamma g'' - 3(v + \alpha + 2a\kappa + 3\gamma\kappa^2)g - (3\lambda + 2\theta)g^3 = 0 \tag{6}$$

To kick off with the analysis of the real part equation, we multiply both sides of (2) by g' and integrate with respect to s to recover

$$2(a + 3\gamma\kappa)(g')^2 = 2(\omega + \alpha\kappa + a\kappa^2 + \gamma\kappa^3)g^2 - (b - \lambda\kappa)g^4, \tag{7}$$

after choosing the integration constant to be zero since the hunt is for a soliton solution. Upon separating variables in (7) and integrating, again with integration constant chosen to be zero, leads to

$$g(s) = A_1\,\mathrm{sech}[B_1(x - vt)] \tag{8}$$

where

$$A_1 = \sqrt{\frac{2(\omega + \alpha\kappa + a\kappa^2 + \gamma\kappa^3)}{b - \lambda\kappa}}, \tag{9}$$

and

$$B_1 = \sqrt{\frac{\omega + \alpha\kappa + a\kappa^2 + \gamma\kappa^3}{a + 3\gamma\kappa}}. \tag{10}$$

Thus, the chirp-free bright 1-soliton solution to the model is:

$$q(x, t) = A_1\,\mathrm{sech}[B_1(x - vt)]e^{i(-\kappa x+\omega t+\theta_0)} \tag{11}$$

where amplitude of the soliton is given by A_1 and inverse width is indicated by B_1. These two parameters introduce the constraints

$$(b - \lambda\kappa)(\omega + \alpha\kappa + a\kappa^2 + \gamma\kappa^3) > 0, \tag{12}$$

and

$$(a + 3\gamma\kappa)(\omega + \alpha\kappa + a\kappa^2 + \gamma\kappa^3) > 0. \tag{13}$$

Similarly, multiplying the imaginary part equation (6) by g' and integrating once with respect to s, while choosing the integration constant to be zero, gives:

$$6\gamma(g')^2 = 6(v + \alpha + 2a\kappa + 3\gamma\kappa^2)g^2 + (3\lambda + 2\theta)g^4. \tag{14}$$

Once again, separating variables and integrating yields:

$$g(s) = A_2\,\mathrm{sech}[B_2(x - vt)] \tag{15}$$