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Dark-singular combo optical solitons with fractional complex Ginzburg–Landau equation

M.A. Abdou[a,b], A.A. Soliman[c,d], Anjan Biswas[e,f], Mehmet Ekici[g], Qin Zhou[h,*], Seithuti P. Moshokoa[f]

[a] Physics Department, College of Science, University of Bisha, Bisha 61922, P.O. Box 344, Kingdom of Saudi Arabia
[b] Theoretical Research Group, Physics Department, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt
[c] Department of Mathematics, College of Science, University of Bisha, Bisha 61922, P.O. Box 344, Kingdom of Saudi Arabia
[d] Department of Mathematics, Faculty of Science, Arish University, Al-Arish 45111, Egypt
[e] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[f] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[g] Department of Mathematics, Faculty of Science and Arts, Bozok University, 66100 Yozgat, Turkey
[h] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China

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ABSTRACT

This paper employs dextended Jacobi's elliptic function expansion method to retrieve doubly periodic function as solutions to the stochastic complex Ginzburg–Landau equation. In the limiting case, when the modulus of ellipticity approaches unity, these solutions approach optical solitons. This paper lists the dark-singular combo optical solitons.

1. Introduction

Optical solitons with fractional temporal evolution as well as fractional group–velocity dispersion is one of the very many interesting areas of development in the study of optical fibers. This paper considers stochastic version of complex Ginzburg–Landau equation (CGLE) to address this issue. There are a variety of mathematical techniques that have been developed to analyze these models with the principles of fractional calculus [1–32]. This fractional calculus has turned out to be the focus of many research activities with a variety of applications and these are from control theory, fluid mechanics, optical fiber technology, plasma physics, solid state physics, mathematical biology, chemical kinematics [27,18,21,28,6,29,10,19]. Several integration schemes were founded to secure the exact traveling wave solutions to a wide range of fractional nonlinear evolution equations (NLEEs) [20,31,22,23,32,24,4,13,11,3,12,2,5,7–9,26,25,15]. The research of optical solitons, in this context, has gained popularity in the field of nonlinear optics. There are enumerable models pertaining to specific situations in nonlinear optical fiber [7–9,26,25,15]. Fractional derivative aid in the description of memory and hereditary properties of materials and processes. Popularly, fractional derivative is defined in the sense of Riemann–Liouville derivative or Caputo derivative although several other forms of derivative do exist.

The Jacobi's elliptic function (JEF) expansion approach for finding periodic wave solutions to NLEEs was proposed in

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

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[1,14,17,16,30]. Essentially, this method involves seeking solution in polynomial form of sn, where sn is the Jacobi's elliptic snoidal function. The aim of this study is to venture further into extended Jacobi's elliptic snoidal function expansion method and apply this scheme to fractional CGLE.

1.1. Governing model

This paper examines the proposed algorithm called extended JEF expansion scheme for retrieving soliton and other solutions to the proposed model given by [5]:

$$i\frac{\partial^\delta u}{\partial t^\delta} + a\frac{\partial^{2\delta} u}{\partial x^{2\delta}} + \mathrm{bF}(|u|^2)u = \frac{1}{|u|^2 u^*}\left[\alpha|u|^2\frac{\partial^{2\delta}|u|^2}{\partial x^{2\delta}} - \beta\left(\frac{\partial^\delta |u|^2}{\partial x^\delta}\right)^2\right] + \gamma u, \tag{1}$$

where $0 < \delta < 1$, gives the order of fractional derivative, x is the spatial variable representing distance along the fiber line while t is the time in its dimensionless form. Next, a, b, α, β and γ are constants. The functional F gives the nonlinear form of the optical fiber under consideration. The coefficients of a and b represent the group velocity dispersion (GVD) and nonlinearity, respectively. A persistent balance between GVD and nonlinearity leads to the formation and sustainment of a soliton. The term with constants α, β and γ arise from the perturbative effects [5]. In particular, γ represents detuning factor.

1.2. Preliminaries of local fractional calculus

The recapitulation of salient properties from modified Riemann–Liouville derivative are presented as follows [19,20]:

$$D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{\mathrm{dt}}\int_0^t (t-\xi)^{-\alpha}(f(\xi)-f(0))d\xi, \quad 0<\alpha<1, \tag{2}$$

$$D_t^\alpha f(t) = (f^n(t))^{\alpha-n}, \quad n<\alpha<n+1, \quad n>1, \tag{3}$$

$$D_t^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)}t^{\gamma-\alpha}, \quad \gamma>0, \tag{4}$$

$$D_t^\alpha(u(t)v(t)) = v(t)D_t^\alpha u(t) + u(t)D_t^\alpha v(t), \tag{5}$$

$$D_t^\alpha[f(u(t))] = f_u'[u(t)]D_t^\alpha u(t), \tag{6}$$

$$D_t^\alpha[f(u(t))] = f_u^\alpha f[u(t)](u_t'(t))^\alpha. \tag{7}$$

2. Mathematical analysis

In order to tackle the fractional CGL Eq. (1), it is utilized the fractional complex transformation given by

$$u(x,t) = U(\xi)e^{i\tau}, \tag{8}$$

where

$$\xi = \frac{x^\delta}{\Gamma(1+\delta)} - \frac{\nu t^\delta}{\Gamma(1+\delta)}, \tag{9}$$

$$\tau = \frac{-kx^\delta}{\Gamma(1+\delta)} + \frac{wt^\delta}{\Gamma(1+\delta)} + \theta. \tag{10}$$

Here, ν is speed of the soliton and k is its frequency with w being the wave number and θ plays the role of phase constant. Inserting (8) into (1) and separating into into real and imaginary parts yield

$$-\mathrm{wU} + a(U'' - k^2U) + \mathrm{bF}(U^2)U = 2(\alpha - 2\beta)\frac{(U')^2}{U} + 2\alpha U'' + \gamma U, \tag{11}$$

and

$$\nu = -2\mathrm{a}\,\mathrm{k}. \tag{12}$$

Up to choosing $\alpha = 2\beta$, for integrability purpose, Eq. (11) is recasted as

$$(a - 4\beta)U'' - (w + ak^2 + \delta)U + \mathrm{b\,F}(U^2)U = 0. \tag{13}$$

2.1. Kerr law

The Kerr law medium, or cubic nonlinearity, is the case when $F(s) = s$, and Eq. (13) reduces to

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