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Landau-Zener tunneling in spin-orbit-coupled Bose-Einstein condensates in bichromatic optical lattices

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ABSTRACT

Landau-Zener transition probability between Bloch bands is carefully investigated in Bose-Einstein condensates belonging to bichromatic optical lattices. The role of the spin-orbit coupling, the energy detuning, the relative phases and depth of primary and secondary lattice are mapped out for the corresponding transition. We find that Landau-Zener tunneling probability strongly depends on the spin-orbit coupling and relative phases of the lattices when the depth of the prime lattice is less than critical values. It is also found that the nonzero transition probabilities mainly occur in the higher Bloch band gaps when the spin-orbit coupling is presented.

1. Introduction

Landau-Zener (LZ) tunneling is a basic quantum process describing the transition dynamics of a two-level system with the energy separation dominated by a time-dependent linear interaction [1,2], and has wide applications in different systems, such as gravity driven atom chains [3], hyperfine and spin-phonon coupling in molecular clusters [4], dynamics of matter waves of Bose-Einstein condensates (BECs) in optical lattices [5], and acoustic waves through a water cavity superlattice [6]. With the experimental realization of BECs in cold atom system, LZ transition problem of original electronic adiabatic energy levels transition by external fields is extended to the Bose system.

The quantum motion of ultracold atoms in an accelerating optical potential is investigated and find the nonlinearity can enhance the tunneling probability between the ground band and the first excited band [7–9]. Moreover, it find that LZ transition between two energy bands is direction-dependent. The probability from lower energy bands to higher bands is different from the tunneling from higher energy bands [5].

Recently, the higher orbital bands in optical lattices have attracted much attention for understanding the various unconventional properties in cold Bose gases [10]. The topologically protected gapless phases of Fermions in higher orbits have also been intensively investigated [11–13]. Since the experimental realization of spin-orbit-coupled BECs with the laser-coupling techniques in ultracold atom gases, the atom orbit momentum coupled pseudo-spin state in higher orbital bands has been the object of intensive theoretical and experimental studies [14–16]. The multiple controllability in the Bose systems provides exclusive opportunities to investigate novel effect in spin-orbit-coupled BECs [17,18]. Especially, the band structures in optical lattices have provided a novel platform for the investigation of the SOC effects in higher orbital energy bands. However many studies in optical lattices largely focus on atoms trapped in the lowest and the first excited bands. For example, the gap solitons in the first and second band gaps of an optical lattice are studied in the spin-orbit-coupled BECs [19], and the lowest Bloch state forms an isolated flat band in a one-dimensional SOC optical lattice [20]. But very few of these activities have been undertaken for the high orbital cases.

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Compared with a simple cubic monochromatic lattices, bichromatic lattices have richer novel effects in the BEC system, for example, wavelengthes of laser, strengthes of lattices and phase difference are all adjustable [21–26]. Experimentally, the bichromatic optical lattice can be formed by adding a second lattice with a different wavelength along one special direction. By adjusting the different physical parameters of the bichromatic superlattice, one can tune the combined lattice of double wells arbitrarily, and control creation of a valence bond state of delocalized effective-spin singlet and triplet dimers [27]. It also can drastically adjust the band structure of the system, consequently affecting the dynamics of the BECs atoms in different Bloch energy band [28]. The ultracold atoms loaded in bichromatic superlattices have been produced many other interesting phenomena as below: subdiffusive transport via disorder and nonlinearity [29], the time-resolved valence-bond quantum resonance [30], and the Anderson localization for matter waves [31,32]. Due to the potential application in many fields, for example, quantum entanglement and strongly correlated many-body phenomena, LZ tunneling in spin-orbit-coupled BECs has attracted intensive attentions.

In this paper, we present a comprehensive analysis of LZ tunneling for the spin-orbit-coupled BECs. Our studies focus on the role of the spin-orbit coupling (SOC), the energy detuning and relative phases. It will be shown that LZ tunneling strongly depends on the SOC strength and relative phase of primary and secondary lattices. The third gap transition probabilities diminish, and the fourth gap transition probabilities increase along the increasing of the detuning between the Raman beam and energy states of the atoms. While the first gap LZ transition probabilities almost keep almost zero for all the cases of phase differences when the SOC strength is bigger than a critical value. The second gap tunneling probabilities all are zero except for the very small SOC in the case of nonzero relative phase differences of the two lattices. The higher orbital band gap tunneling probabilities may form the Gaussian or double well distribution according to the phase differences for the different SOC and detuning strength. The strengthes of the primary and secondary lattices also play significant roles on the higher orbital band gap tunneling probabilities, which all decrease with the increasing of the primary lattice heights. On the contrary, the tunneling behaviors occur oscillating waveform as the secondary lattice amplitudes increase for the parameter regime.

Our paper is organized as follows. In Section 2, we introduce the model equation for LZ tunneling problem in a spin-orbit-coupled Bose system. Section 3 is devoted to compute the different LZ tunneling probabilities for the SOC strengths, the depth of the primary and secondary lattices and phase differences in the case of zero detuning. It is aimed to give a direct insight of the different tunneling probabilities in an interval of parameters. In Section 4, we calculate the tunneling rate for various related parameters in the finite detuning regimes. In Section 5, we draw our conclusions and make a brief summary.

2. Theoretical model

When the transversal freedom of BEC is frozen because larger transversal confined frequencies compare with longitudinal trapping frequency, the motion of spin-orbit-coupled BEC can be modeled by the one-dimensional time-dependent coupled nonlinear Schrödinger equations in the mean-field levels as

$$i\hbar\frac{\partial\Psi}{\partial t}= \left[-\frac{1}{2m}\left(\hbar\frac{\partial}{\partial x}-\mathrm{i}ma_l t\right)^2 - i\gamma\frac{\partial}{\partial x}\sigma_z+\delta\sigma_z+\Omega\sigma_x\right]\Psi + V_{\mathrm{trap}}\Psi+\frac{4\pi\hbar^2 a_s}{m}(|\Psi_1|^2+|\Psi_2|^2)\Psi, \tag{1}$$

where the spinor wave function $\Psi = (\Psi_1, \Psi_2)^T$ and Ψ_1, Ψ_2 is the up or down pseudo-spin component respectively. Here the $\{\sigma_i\}$ are the Pauli matrices, m is the atom mass. A force of ma_l is represented in the vector potential gauge, which can be represented by inertial force in the frame of an accelerating lattice and emulated an electric force on Bloch electrons in solid state materials. The SOC happens in the x direction and the coefficient $\gamma = \hbar\, k_{Ram}/m$ is proportional to the wavenumber of the Raman beams k_{Ram}, Ω is Raman coupling strength between the two pseudo-spin energy levels, δ is the energy detuning between the Raman beams. The external trap is $V_{trap} = \mathrm{v}_1\cos(2k_L x) + \mathrm{v}_2\cos(4k_L x + \theta)$, where k_L is the wavenumber of lattice beams, θ is the phase difference of primary and secondary lattices. We will assume $a_s = a_{s_{11}} = a_{s_{22}} = a_{s_{12}}$, the parameters $a_{s_{11}}$ and $a_{s_{22}}$ are the s-wave scattering lengthens between atoms within each component, while $a_{s_{12}}$ represents interactions between the two components atoms. The difference between the s-wave scattering lengths in the corresponding experiments is very small and the above equivalent relationship is appropriate. To find Bloch bands numerically, we start from the dimensionless coupled nonlinear Schrödinger equations

$$i\hbar\frac{\partial\Phi}{\partial t}= \left[-\frac{1}{2}\left(\frac{\partial}{\partial x}-\mathrm{i}a_l t\right)^2 - i\gamma\frac{\partial}{\partial x}\sigma_z+\delta\sigma_z+\Omega\sigma_x\right]\Phi + (\mathrm{v}_1\cos(x)+\mathrm{v}_2\cos(2x+\theta))\Phi+c(|\Phi_1|^2+|\Phi_2|^2)\Phi, \tag{2}$$

where the units of length, time and energy are $1/(2k_{lat})$, $\hbar/\mathrm{E}_{\mathrm{lat}}$, $\mathrm{E}_{\mathrm{lat}} = 8\mathrm{E}_{\mathrm{rec}}$, $\mathrm{E}_{\mathrm{rec}} = \hbar^2 k_{\mathrm{lat}}^2/(2m)$ is recoil energy of lattce, and the nolinear coefficient $c = \pi n_0 a_s/k_{\mathrm{lat}}^2$, respectively. The accelerated speed of lattice a_l is in the unit of $8\hbar^2 k_{\mathrm{lat}}^3/m^2$. The dimensionless wavefunction is given by $\Phi = \Psi/\sqrt{n_0}$, so that the number density of atoms n_0 turns out as

$$n_0 = \frac{1}{L}\int \mathrm{d}x(|\Phi_1|^2+|\Phi_2|^2), \tag{3}$$

the dimensionless SOC parameter $\gamma = k_{Ram}/(2k_{lat})$. The Raman coupling strength Ω, the energy detuning δ, the depth of primary and secondary lattices v_1 and v_2 are all in the unite of energy $\mathrm{E}_{\mathrm{lat}}$.

The dimensionless coupled nonlinear Schrödinger equations Eq. (2) have the stationary solution of the form $\Phi(x, t) = \phi(x)\exp(-i\mu t)$, with μ being the chemical potential. The energy μ distributes along with the moment k according to the different band indices