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Chirp-free bright optical soliton perturbation with Fokas–Lenells equation by traveling wave hypothesis and semi-inverse variational principle

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ABSTRACT

This paper secures chirp-free bright optical solitons to perturbed Fokas–Lenells equation. Two integration algorithms are applied. They are traveling wave hypothesis and the semi-inverse variational principle. The parameter restrictions for the existence of solitons are also presented.

1. Introduction

The study of optical soliton perturbation has indeed come a long way through since its first appearance. In the past, there were only a few integration algorithms available at our disposal to study and retrieve soliton or any other solutions from a given nonlinear evolution equation (NLEE). Inverse Scattering Transform (IST) and Hirota's bilinear method were two of the very few and powerful mechanisms that we depended upon. The story is very different today. There is a wide range of tools available at our disposal. This has eased off the integration frustration that lingered on during those days. IST was not applicable to all forms of NLEE. For example, when the governing nonlinear Schrödinger's equation (NLSE) is with power law or dual-power law or anti-cubic law, it fails to integrate NLSE and extract soliton solutions to the model. This led to the discovery of a variety of integration algorithms that we enjoy today [1–25]. This paper will apply two of the integration schemes. One is the classic traveling wave hypothesis while the second is the semi-inverse variational principle (SVP) that exists for about a decade. This paper studies Fokas–Lenells equation (FLE) with a few perturbation terms included. FLE stems from NLSE when group velocity dispersion (GVD) is negligibly small. In this case, additional dispersion terms must be included to compensate for low GVD so that the delicate balance between dispersion and nonlinearity is maintained for solitons to sustain. The details are explored in the rest of the paper.

1.1. Governing model

The perturbed FLE in its dimensionless form is of the form [6–10]:

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$$iq_t + a_1 q_{xx} + a_2 q_{xt} + |q|^2(bq + i\sigma q_x) = i[\alpha q_x + \lambda(|q|^2 q)_x + \theta(|q|^2)_x q] \tag{1}$$

where $q(x, t)$ is a complex-valued function that represents the soliton profile and $i = \sqrt{-1}$. The independent variables are x and t that stand for spatial and temporal variables respectively. The coefficients of a_j for $j = 1, 2$ represent group velocity dispersion (GVD) and spatio-temporal dispersion (STD) respectively. While b is the coefficient of Kerr law nonlinearity, the coefficient of σ is the additional dispersion terms that provides the necessary balance between dispersion and nonlinearity for the solitons to sustain whenever GVD and/or STD carry a low count. The right hand side appears when perturbative terms creep in during the derivation of the model from Maxwell's equation. From these terms on the right hand side, α is the coefficient of inter-modal dispersion that is needed in addition to chromatic dispersion, while λ is the effect of self-steepening that avoids the formation of shock waves and finally θ provides the effect of nonlinear dispersion. The model thus introduced, this paper will now embark into the retrieval of bright soliton solutions using two procedures. They are traveling wave hypothesis and SVP. These are explored in the rest of the paper.

2. Mathematical analysis

In order to initiate the integration process, the start-up assumption is [6–10]:

$$q(x, t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where

$$s = x - vt, \tag{3}$$

and v is the speed of the soliton. The phase component of the pulse is structured as:

$$\phi(x, t) = -\kappa x + \omega t + \theta_0. \tag{4}$$

Here, κ is the soliton frequency, ω is the soliton wave number and θ_0 is the phase constant.

Next, substitute (2) into (1) and then split into real and imaginary parts. The real part yields:

$$(a_1 - a_2 v)g'' - (\omega + \alpha\kappa + a_1\kappa^2 - a_2\kappa\omega)g + (b + \sigma\kappa - \lambda\kappa)g^3 = 0 \tag{5}$$

where $g' = dg/ds$, $g'' = d^2g/ds^2$ and so on. Then, the imaginary part gives:

$$v + \alpha + 2a_1\kappa - a_2(v\kappa + \omega) + (3\lambda + 2\theta - \sigma)g^2 = 0. \tag{6}$$

From (6), velocity of the soliton falls out to be

$$v = \frac{\alpha + 2a_1\kappa - a_2\omega}{a_2\kappa - 1} \tag{7}$$

and the constraint condition that emerges is:

$$3\lambda + 2\theta - \sigma = 0. \tag{8}$$

The restriction that (7) implies is

$$a_2\kappa \neq 1. \tag{9}$$

The real part equation, given by (5) will be now analyzed in the subsequent two subsections.

2.1. Traveling wave hypothesis

Multiplying both sides of (5) by g' and integrating gives

$$2(a_1 - a_2 v)(g')^2 = 2(\omega + \alpha\kappa + a_1\kappa^2 - a_2\kappa\omega)g^2 - (b + \sigma\kappa - \lambda\kappa)g^4 \tag{10}$$

where the integration constant is chosen to be zero keeping in mind that soliton solutions are being retrieved for the model. Separating variables and integrating, with zero constant of integration, leads to

$$g(s) = A\,\mathrm{sech}[B(x - vt)] \tag{11}$$

where

$$A = \sqrt{\frac{2(\omega + \alpha\kappa + a_1\kappa^2 - a_2\kappa\omega)}{b + \sigma\kappa - \lambda\kappa}}, \tag{12}$$

and

$$B = \sqrt{\frac{\omega + \alpha\kappa + a_1\kappa^2 - a_2\kappa\omega}{a_1 - a_2 v}}. \tag{13}$$

Thus, the chirp-free bright 1-soliton solution to the model is:

$$q(x, t) = A\,\mathrm{sech}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta_0)} \tag{14}$$