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Optical solitons with differential group delay and dual-dispersion for Lakshmanan–Porsezian–Daniel model by extended trial function method

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ABSTRACT

This paper employs extended trial function method to extract soliton solutions to the vector-coupled Lakshmanan–Porsezian–Daniel model in birefringent fibers. The existence of these solitons is guaranteed with the integrability criteria that are also presented.

1. Introduction

Optical soliton perturbation is one of the fastest growing areas of research in the field of telecommunications industry. The engineering marvel in this field as it stands owes to several mathematical models that are studied in this context [1–20]. These plethora of models describe the dynamics in a variety of situations and circumstances. These models include the most visible non-linear Schrödinger's equation, Schrödinger-Hirota equation for dispersive solitons, Manakov equation for polarization mode dispersion (PMD), Sasa-Satsuma equation for perturbed solitons and several others. This paper will address soliton study for PMD fibers with differential group delay with Lakshmanan–Porsezian–Daniel (LPD) model. It's scalar version is well studied and is well-known and well understood [1,3–8,13,14,16,19,20]. It's about time to turn the page for this model. The vector coupled version of LPD model to address birefringent fibers, without four-wave mixing (4WM) have been reported [2]. This paper will revisit the model for PMD fibers by the aid of extended trial function method. After a quick introduction to the system, with preliminary mathematical analysis, the scheme will retrieve soliton solutions. The integrability criteria will also be presented that guarantees the existence of these solitons and other such waves.

1.1. Governing model

The dimensionless form of LPD model, with Kerr law nonlinearity, that has been studied in the past is given in the form

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[1,3–8,13,14,16,19,20]:

$$iq_t + aq_{xx} + bq_{xt} + c\,|q|^2q = \sigma q_{xxxx} + \alpha(q_x)^2q^* + \beta\,|q_x|^2q + \gamma\,|q|^2q_{xx} + \lambda q^2q_{xx}^* + \delta\,|q|^4q. \tag{1}$$

In (1), the real-valued coefficients a and b represent group velocity dispersion and spatio-temporal dispersion respectively. Then, c is the coefficient of Kerr law nonlinearity and σ is the fourth order dispersion while δ accounts for two-photon absorption. The remaining terms are from other forms of dispersive phenomenon [3]. Soliton formation is the outcome of a delicate balance between dispersive and nonlinear effects.

For birefringent fibers, the model therefore splits into two components leading to the coupled vector form of LPD. After neglecting the effects of 4WM, this coupled system takes the form [2]:

$$\begin{aligned} & iq_t + a_1q_{xx} + b_1q_{xt} + (c_1\,|q|^2 + d_1\,|r|^2)q \\ = \; & \sigma_1q_{xxxx} + (\alpha_1q_x^2 + \beta_1r_x^2)q^* + (\gamma_1\,|q_x|^2 + \delta_1\,|r_x|^2)q \\ + \; & (\lambda_1\,|q|^2 + \theta_1\,|r|^2)q_{xx} + (\xi_1q^2 + \eta_1r^2)q_{xx}^* + (f_1\,|q|^4 + g_1\,|q|^2|r|^2 + h_1\,|r|^4)q, \end{aligned} \tag{2}$$

$$\begin{aligned} & ir_t + a_2r_{xx} + b_2r_{xt} + (c_2\,|r|^2 + d_2\,|q|^2)r \\ = \; & \sigma_2r_{xxxx} + (\alpha_2r_x^2 + \beta_2q_x^2)r^* + (\gamma_2\,|r_x|^2 + \delta_2\,|q_x|^2)r \\ + \; & (\lambda_2\,|r|^2 + \theta_2\,|q|^2)r_{xx} + (\xi_2r^2 + \eta_2q^2)r_{xx}^* + (f_2\,|r|^4 + g_2\,|r|^2|q|^2 + h_2\,|q|^4)r. \end{aligned} \tag{3}$$

Here, in (2) and (3), c_j and f_j ($j = 1, 2$) account for self-phase modulation while the coefficients of d_j, g_j and h_j stem from cross-phase modulational effect.

2. Mathematical analysis

In order to tackle the governing coupled system, the starting hypothesis is selected in the form:

$$q(x, t) = P_1[\eta(x, t)]\exp[i\phi(x, t)], \tag{4}$$

$$r(x, t) = P_2[\eta(x, t)]\exp[i\phi(x, t)], \tag{5}$$

where $P_l(\eta)$ for $l = 1, 2$ are the amplitude component of the soliton and

$$\eta = x - \mathrm{v}\,\mathrm{t}, \tag{6}$$

and the phase component ϕ is defined as

$$\phi = -\kappa x + \omega t + \theta, \tag{7}$$

for $l = 1, 2$. Here, v is the velocity of the soliton, κ is the frequency of the solitons in each of the two components while ω is the soliton wave number and θ is the phase constant. Inserting (4) and (5) into (2) and (3) and decomposing into real and imaginary parts yields, respectively

$$\begin{aligned} & (\omega + a_l\kappa^2 - b_l\kappa\omega + \kappa^4\sigma_l)P_l - (c_l + \kappa^2(\alpha_l - \gamma_l + \lambda_l + \xi_l))P_l^3 + f_lP_l^5 \\ & - (d_l + \kappa^2(\beta_l - \delta_l + \eta_l + \theta_l))P_lP_{\bar{l}}^2 + g_lP_l^3P_{\bar{l}}^2 + h_lP_lP_{\bar{l}}^4 + (\alpha_l + \gamma_l)P_l(P_l')^2 \\ & + (\beta_l + \delta_l)P_l(P_{\bar{l}}')^2 - (a_l - b_lv + 6\kappa^2\sigma_l)P_l'' + (\lambda_l + \xi_l)P_l^2P_l'' + (\eta_l + \theta_l)P_{\bar{l}}^2P_l'' + \sigma_lP_l^{(4)} = 0, \end{aligned} \tag{8}$$

and

$$(v + 2a_l\kappa - b_l(v\kappa + \omega) + 4\kappa^3\sigma_l)P_l' - 2\kappa(\alpha_l + \lambda_l - \xi_l)P_l^2P_l' + 2\kappa(\eta_l - \theta_l)P_{\bar{l}}^2P_l' - 2\beta_l\kappa P_lP_{\bar{l}}P_{\bar{l}}' - 4\kappa\sigma_lP_l^{(3)} = 0, \tag{9}$$

for $l = 1, 2$ and $\bar{l} = 3 - l$. The balancing principle gives

$$P_{\bar{l}} = P_l, \tag{10}$$

and then, Eqs. (8) and (9) become

$$\begin{aligned} & (\omega + a_l\kappa^2 - b_l\kappa\omega + \kappa^4\sigma_l)P_l - (c_l + d_l + \kappa^2(\alpha_l + \beta_l - \gamma_l - \delta_l + \eta_l + \theta_l + \lambda_l + \xi_l))P_l^3 \\ & + (f_l + g_l + h_l)P_l^5 + (\alpha_l + \beta_l + \gamma_l + \delta_l)P_l(P_l')^2 - (a_l - b_lv + 6\kappa^2\sigma_l)P_l'' \\ & + (\eta_l + \theta_l + \lambda_l + \xi_l)P_l^2P_l'' + \sigma_lP_l^{(4)} = 0, \end{aligned} \tag{11}$$

and

$$(v + 2a_l\kappa - b_l(v\kappa + \omega) + 4\kappa^3\sigma_l)P_l' - 2\kappa(\alpha_l + \beta_l - \eta_l + \theta_l + \lambda_l - \xi_l)P_l^2P_l' - 4\kappa\sigma_lP_l^{(3)} = 0, \tag{12}$$

respectively. From (12), the third term needs

$$\sigma_l = 0, \tag{13}$$

for $l = 1, 2$. This means that soliton solutions to (2) and (3), will exist provided fourth order dispersion vanishes. The remaining linearly independent functions, from Eq. (12), give rise to the constraints