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Optical solitons in birefringent fibers for Lakshmanan–Porsezian–Daniel model using $\exp(-\phi(\xi))$-expansion method

Saima Arshed[a], Anjan Biswas[b,c,d], Fayequa B. Majid[b], Qin Zhou[e,*], Seithuti P. Moshokoa[d], Milivoj Belic[f]

[a] Department of Mathematics, University of the Punjab, Lahore 54590, Pakistan
[b] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[c] Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh 13318, Saudi Arabia
[d] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[f] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

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ABSTRACT

This paper studies optical solitons in birefringent fibers by the aid of exp-function approach. The model that is analyzed is Lakshmanan–Porsezian–Daniel equation which is first written in two-component form for vector solitons. The integrability criteria are also presented in the paper.

1. Introduction

The theory of optical solitons has drastically advanced the telecommunications industry with captivating technological marvel. There has been several improvements from mathematical and technological perspectives that advanced this industry with cutting edge technology. With a plethora of mathematical models that has near-perfected the dynamics of soliton science in optical fibers and metamaterials, there is still a lot to explore. This paper studies soliton dynamics with Lakshmanan–Porsezian–Daniel (LPD) model in birefringent fibers with differential group delay. The model equations are obtained from LPD equation that is visible in polarization-preserving fibers. The effect of four-wave mixing (4WM) is not included to keep the model simple. The vector coupled LPD equation describing birefringence is integrated, in this paper, by the aid of $\exp(-\phi(\xi))$-expansion scheme. It must be noted that there are several forms of integration algorithms that are utilized to retrieve soliton solutions and carry out analytical studies in nonlinear fiber optics [1–15]. Here, in order to integrate the governing model, phase-matching condition is implemented. The details of the mathematical principles are first enumerated followed by the derivation and classification of soliton solutions.

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

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2. Governing model

The dimensionless form of LPD model, with Kerr law nonlinearity, that has been studied in the past is given in the form [2–10,13–15]:

$$iq_\tau + aq_{xx} + bq_{x\tau} + c|q|^2 q = \sigma q_{xxxx} + \alpha(q_x)^2 q^* + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta|q|^4 q \tag{1}$$

In (1), the real-valued coefficients a and b represent group velocity dispersion and spatio-temporal dispersion respectively. Then, c is the coefficient of Kerr law nonlinearity and σ is the fourth order dispersion while δ accounts for two-photon absorption. The remaining terms are from other forms of dispersive phenomenon [2–10,13–15]. Soliton formation is the outcome of a delicate balance that sustains between dispersive and nonlinear effects.

For birefringent fibers, the model therefore splits into two components leading to the coupled vector form of LPD. After neglecting the effects of 4WM, this coupled system is [1]:

$$\begin{aligned} & iq_\tau + a_1 q_{xx} + b_1 q_{x\tau} + (c_1|q|^2 + d_1|r|^2)q \\ = {} & \sigma_1 q_{xxxx} + (\alpha_1 q_x^2 + \beta_1 r_x^2)q^* + (\gamma_1|q_x|^2 + \delta_1|r_x|^2)q \\ + {} & (\lambda_1|q|^2 + \theta_1|r|^2)q_{xx} + (\xi_1 q^2 + \eta_1 r^2)q^*_{xx} + (f_1|q|^4 + g_1|q|^2|r|^2 + h_1|r|^4)q \end{aligned} \tag{2}$$

$$\begin{aligned} & ir_\tau + a_2 r_{xx} + b_2 r_{x\tau} + (c_2|r|^2 + d_2|q|^2)r \\ = {} & \sigma_2 r_{xxxx} + (\alpha_2 r_x^2 + \beta_2 q_x^2)r^* + (\gamma_2|r_x|^2 + \delta_2|q_x|^2)r \\ + {} & (\lambda_2|r|^2 + \theta_2|q|^2)r_{xx} + (\xi_2 r^2 + \eta_2 q^2)r^*_{xx} + (f_2|r|^4 + g_2|r|^2|q|^2 + h_2|q|^4)r \end{aligned} \tag{3}$$

Here, in (2) and (3), c_j and f_j $(j = 1, 2)$ represent coefficients of self-phase modulation while the coefficients of d_j, g_j and h_j are accounted for cross-phase modulation.

3. Mathematical analysis

This section will be first a revisitation of the proposed analytical scheme followed by its applicability to the LPD model with birefringence. The details are penned down in the following two subsections.

3.1. Proposed analytical technique

Let us take a look at the following general nonlinear evolution equation (NLEE)

$$P(u, D_\tau u, D_x u, D_\tau^2 u, D_{x\tau} u, D_x^2 u, \ldots) = 0, \tag{4}$$

where $u = u(x, \tau)$ is an unknown function and P is a polynomial in u and its partial derivatives, in which the nonlinear terms and its highest order derivatives are taken into consideration. The traveling wave solution can be recovered with the wave transformation that reads:

$$u(x, \tau) = U(\xi), \quad \xi = k(x - v\tau).$$

After implementing this wave transformation, the NLEE is converted into a nonlinear ordinary differential equation (ODE) given as:

$$S(U, U', U'', U''', \ldots) = 0, \tag{5}$$

where $'$ denotes the derivative with respect to ξ.

Based on $\exp(-\phi(\xi))$-expansion method, the solution can be casted as:

$$U(\xi) = \sum_{n=0}^{N} a_n(\exp(-\Phi(\xi)))^n, \tag{6}$$

where a_n are unknown constants that are to be determined and $\Phi(\xi)$ governs the auxiliary ODE given by:

$$\Phi'(\xi) = \exp(-\Phi(\xi)) + \mu\exp(\Phi(\xi)) + \lambda. \tag{7}$$

The auxiliary Eq. (7) lists the general family of solutions as:

Case 1. (Hyperbolic function solutions) If $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, then

$$\Phi_1(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + C)\right) - \lambda}{2\mu}\right). \tag{8}$$

Case 2. (Trigonometric function solutions) However, if $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$,