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Original research article

Use of fractional operator to define intermediate zones in electromagnetics via Dirichlet Green's function

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ABSTRACT

The principal goal of this paper is to investigate the intermediate zones in electromagnetics when a radiation/scattering problem is modeled through Dirichlet Green's function. Field/potential quantities over two parallel observation flat planes are related by an integral transform with K as its kernel. The kernel of the integral transform may be divided into two parts: one is K^m yielding contribution due to actual source and the other is K^n yielding image contribution. The fractional kernel K_ν with fractional parameter ν can provide a unique way of interpreting the field in the intermediate zone. Moreover, use of this fractional kernel to study the Fresnel and Fraunhofer diffraction zones is also described. Result for corresponding static problem is obtained by applying the limit on this kernel.

1. Introduction

It is believed that fractional calculus was discovered on 30th September 1695, when L' Hospital's wrote a letter to Leibniz asking him about result for a derivative of a linear function $\frac{d^n f(x)}{dx^n}$, when $n = 1/2$ [1]. Fractional calculus is a branch of mathematics that deals with non-integer (real or even complex) fractional order derivatives/integrals [2–12]. Engheta applied the tools of fractional calculus on different problems and derived conclusion that these tools are useful in electromagnetics [13–17]. The tools of fractional calculus have wide range of applications in different subjects. For example, introduction of fractional order TID controllers in control system [18], and description of physical behavior of nuclear reactor dynamics [19]. Applications can also be found in material science, condensed matter [20] and random walk on fractal structure and percolation [21]. Fractional calculus has also been used to derive the analytical solutions for transient viscous-diffusion equation [22] and to suppress chaotic oscillations [23].

When a linear operator – operating on a vector function – is fractionalized, the process corresponds to fractionalization of the eigenvalues of the linear operator. It is worth mentioning that the fractional operator may provide fractional or interpolated solutions between the two given solutions. It may be used to understand the behavior of field/potential in the intermediate situations.

Image theory provides an easy method of finding the solution for geometries involving interfaces. In this theory, interfaces in the geometry can be replaced by taking into account the equivalent sources, also known as image sources. In 1848, Lord Kelvin introduced the method of images [24], which is commonly used to derive potential/field in electromagnetics and in other disciplines. Solution for a point charge in an anisotropic half space which is bounded by an anisotropic surface was proposed by Hanninen et al. using image theory [25]. The electrostatic image theory for the dielectric sphere was treated by Lindell [26] while the solution for two intersecting conducting spheres was also derived by Lindell et al. [27].

Green's function second identity is given by [28]

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$$\oint_S \left(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right)\mathrm{ds} = \int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\mathrm{dv} \tag{1}$$

Setting $\phi = \psi(x', z_0)$ and $\psi = G(x, z;x', z_0)$, the resultant expression is given by

$$\psi(x, z_1) = \int_{-\infty}^{+\infty}\left\{\psi(x', z_0)\frac{\partial G(x, z; x', z_0)}{\partial z}|_{z=z_1} - G(x, z_1; x', z_0)\frac{\partial\psi(x', z_0)}{\partial z}\right\}\mathrm{dx}' \tag{2}$$

Assuming that Green's function satisfies Dirichlet boundary condition, that is $G(x, z_1;x', z_0) = 0$, the above equation can be written as,

$$\psi(x, z_1) = \int_{-\infty}^{+\infty}\psi(x', z_0)\frac{\partial G(x, z_1; x', z_0)}{\partial z}\mathrm{dx}' \tag{3}$$

It is obvious from the above integral expression that the quantities at one observation plane may be related to corresponding quantities at another observation plane. In this manuscript, the observation of radiation due to a two dimensional electric current source has been done at a point between two parallel flat planes. For this purpose, Dirichlet Green's function in Green's second identity is used. In geometry, to satisfy Dirichlet boundary condition, a PEC planar interface is placed. It may be considered an extension to work reported in [17]. Moreover, this is one of the important geometry in antenna theory and scattering/radiation problems. Here two parallel observation planes have been considered and by fractionalization of the kernel of the integral transform, given in (3), the appropriate expressions have been derived to make the observations at locations between the two given parallel planes which is termed intermediate zone.

The purpose is to introduce the fractional operator for this particular geometry satisfying Drichlet boundary condition. Using fractional operator, an analysis has been done to obtain field/potential in the intermediate zone for a specific and important geometry in the radiation/scattering problem. Usually, near and far zone radiated fields are described in the literature. The intermediate zone can also be of interest to research community. This work provides a new way to obtain field in the intermediate zone using this simple analysis. In addition to antenna theory, this analysis may also be helpful in inverse scattering theory to approximate field in intermediate zone. The description also involves operators like those in inverse problems. In this way, in addition to use of field data in near and far zone, data in the intermediate zone can also be used.

2. Geometry and conceptualization of the problem

Consider the radiation geometry as shown in Fig. 1. It contains a two dimensional electric current source which is placed at $J(x', z')$. The observations of field/ potential produced due to the source are made in cartesian coordinate system. It may be noted that field/potential must be independent of y-coordinate as current distribution on the source is independent of y-coordinate. Potential or component of field quantity has been symbolically represented as $\psi(x, z)$ and must satisfy the source free Helmholtz's equation $\nabla^2\psi(x, z) + k_0^2\psi(x, z) = 0$. Two observation planes situated at $z = z_0 > 0$ and $z = z_1 > z_0$ are assumed. It may be noted that observation plane at $z = z_0$ may correspond to the near zone of the source, while $z = z_1$ may correspond to the far-zone of the source. Potential distribution at $z = z_0$ and $z = z_1$ are expressed as $\psi(x, z_0)$ and $\psi(x, z_1)$, respectively. Using the method of images, the Green's function for the geometry may be written as,

$$\begin{aligned} G(x, z; x', z') &= \tfrac{i}{2}H_0^{(1)}(k_0\sqrt{(x-x')^2 + (z+z')^2}) \\ &\quad - \tfrac{i}{2}H_0^{(1)}(k_0\sqrt{(x-x')^2 + (z-z')^2}) \end{aligned} \tag{4}$$

where

$$H_0^{(1)}(k_0\sqrt{(x-x')^2 + (z-z')^2}) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{1}{k_z}\exp\{\mathrm{i}k_x(x-x') + \mathrm{i}k_z(z-z_1)\}\mathrm{dk}_x \tag{5}$$

It is obvious from the above result that $G(x, 0;x', z') = 0$. Therefore Green's function given in (4) may be called Dirichlet Green's function. Two potentials $\psi(x, z_0)$ and $\psi(x, z_1)$ may be related through the following integral transform [17]

$$\psi(x, z_1) = \int_{-\infty}^{+\infty}K(x, z_1; x', z_0)\psi(x', z_0)\mathrm{dx}' \tag{6}$$

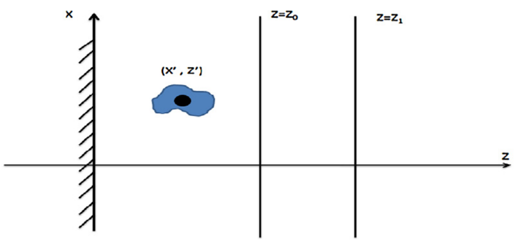

Fig. 1. Geometry of the problem for the two-dimensional wave propagation.