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Image segmentation and selective smoothing based on p-harmonic Mumford–Shah functional

Shuaijie Li[a,*], Peng Li[b]

[a] School of Mathematics and Statistics, Henan University of Science and Technology, Luoyang 471023, Henan, PR China
[b] School of Mathematics and Statistics Science, Ludong University, YanTai, ShanDong 264025, PR China

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ABSTRACT

In this work, we propose a p-harmonic Mumford–Shah (MS) functional with adaptive variable exponent $1 \le p(x) \le 2$ according to image gray feature, which provides a model for image segmentation and smoothing. The paper analyzes the physical characteristics of the related p-harmonic equation in local coordinates and explains that diffusion behavior of p-harmonic is superior to that of anisotropic diffusion and isotropic diffusion in essence. Thus the proposed model is more suitable for segmentation and smoothing of noisy images with intensity inhomogeneities while simultaneously preserving edges than the piecewise smooth MS (PSMS) model. Then effective numerical scheme is constructed to handle its computation using level set method. The model is finally applied on a wide variety of image segmentation and smoothing. All these results show that the proposed model is effective.

1. Introduction

Image segmentation and smoothing are two popular problems and foundational tasks in image processing and computer vision, and recently have achieved great improvement.

Active contours or snakes [1] is used to segment objects automatically, but some potential problems with the approach are that the method does not handle changes of topology, and the initial curve has to surround the objects to be detected. To overcome these problems, level set method, originally introduced by Osher and Sethian [2], has been developed into one of the most successful tools for many practical applications, such as the computation of evolving geometries, detection of the objects interfaces using the zero level sets. The edge-based level set methods [3–5] usually depend on the gradient of the given image, as stopping condition of the evolution of the curve. But it may not detect the true edges because of the noisy effect. Recently, the region-based level set methods [6–9] have been proposed and applied to image segmentation by incorporating region-based information into the energy functional. They can obtain a better performance of segmentation than that of the edge-based level set methods, especially for images with weak object edges and noise. For instance, Chan and Vese [6] proposed a variational model called piecewise constant Mumford–Shah (PCMS) model or CV model based on Mumford–Shah functional [10], where the image was modeled as a piecewise constant function. They successfully solved the minimization problem using level set functions, which utilized the global image statistics inside and outside the evolving curve rather than the gradients on the boundaries. However, its some intrinsic limitations still exist, and often lead to poor segmentation results for images with intensity inhomogeneity. After that, many efficient implementation schemes have been

* Corresponding author.
E-mail address: lsjiesysu@163.com (S. Li).

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proposed [11–16]. Chan and Vese [11] extended the model to segment image using multi-phase level sets to detect multiple objects. Nevertheless, the involved computation is very expensive, which also limits its applications in practice. In addition, to reduce the computational cost, this methods usually require that the initial contour should be near to the object edges. To reduce the computational load of curve evolution for the PCMS model, the implementation schemes without solving the PDEs were proposed [14,15]. Li et al. [16] presented a so-called penalizing energy which acts as a metric to characterize how close the level set function to a signed distance function. This metric can also be adopted by the PCMS model to avoid the re-initialization step. Also Lie et al. [17–19] proposed a piecewise constant level set method (PCLSM), they solved the segmentation problem in a different way, that is, by introducing a piecewise constant level set function, instead of using the zero level of a function to represent the interface between subdomains. Song and Li have already improved the method in [20,21].

In order to segment images with intensity inhomogeneities, Chan and Vese [11,22] showed how the piecewise smooth MS segmentation problem could be solved using the level set method, and they had given the piecewise smooth optimal approximations of a given image. Although the piecewise smooth MS (PSMS) model works better, the convergence of the curve to object edge will be too slow. In [23,24], Zhang improved the algorithm for the PSMS model. Recently, following applications of non-local mean method in image processing [25–30], a novel segmentation approach named non-local active contours [NLAC] was proposed based on non-local mean method and level set function in [28–30], segmented regions with smoothly varying intensity and complicated textures very well, and became a focus in recent researches. In addition, now there also exist some optimization methods to solve the MS model based on level set method, such as Graph cuts [31–33] and AOS (Additive Operator Splitting) [34–36].

Image smoothing based on PDEs has become an active field. Particularly the anisotropic diffusion model originally was introduced by Perona and Malik [37] and further developed such as [38,39]. Besides the total variation model (the ROF model) proposed by Rudin, Osher and Fatemi in [40], has been extensively studied and proven to be efficient for preserving edges [41,42]. However, the ROF model often causes staircase effects [43], and develops false edges that do not exist in images. In the recent decades, high-order PDEs (typically, fourth order PDEs) have been introduced in image restoration [44–46]. The theoretical analysis in [47,48] shows that fourth-order equations have advantages over second-order equations in some aspects. Besides a combinable model with second and fourth order PDEs was proposed in [49,50]. But the models cause the computation complexity and increase the computational expense. Other methods such as p-harmonic model proposed and studied in [51–53] perform better in image smoothing.

Segmentation and smoothing in medical images have very important applications in diagnosis, surgical planning, and medical image analysis. For example, some important features such as edges are very useful for medical images. It is necessary to look for well suited segmentation and smoothing methods that can preserve important features for medical images. The PCMS model [6] is useless for some images with intensity inhomogeneities, but the PSMS model [11,22] is a good method because of its activity of smoothing and segmentation. A problem is that L^2 norm of the gradient in MS model is too strong in requirement of smoothing. Thus the L^2 norm of the gradient allows to remove the noise, but leads to blurring the edges and can't accurately detect objects.

Therefore, the paper proposes a so called p-harmonic MS functional based on previous works, which provides a model for image segmentation and smoothing. We use the L^p norm instead of the L^2 norm in the MS model, where $1 \le p \le 2$ is adaptive variable exponent according to image gray feature, and we design effective numerical scheme to minimize the functional using level set method. Particularly, it can be found that our model is more suitable for noisy image segmentation and smoothing while simultaneously preserving edges in experiments of some medical images with intensity inhomogeneities.

The outline of the paper is as follows. In Section 2, we review briefly the MS model and the PSMS model. And we propose p-harmonic MS functional and analyze its performance of smoothing as a model in Section 3. Section 4 discusses some numerical implementation issues in detail, and experiments and conclusion are drawn in the last two sections.

2. Preliminary works

2.1. Mumford–Shah model

The MS model is a variational model for approximating a given image by a piecewise smooth image of minimal complexity. Let $\Omega \in R^N$ ($N = 2$ in our consideration) be an open and bounded domain with Lipschitz boundary, modeling the image domain. Let $u_0 : \Omega \to R$ be a bounded function, representing a grayscale image. To find the segmentation C of u_0, D. Mumford and J. Shah proposed the following minimizing problem of piecewise smooth segmentation [10]:

$$\inf_{u,C}\{F^{MS}(u, C) = \int_{\Omega} |u - u_0|^2 dx + \mu \int_{\Omega \setminus C} |\nabla u|^2 dx + \nu |C|\}, \tag{1}$$

where $\mu > 0$, $\nu > 0$ are nonnegative constants, to weight the different terms in the energy. A minimizer of the above energy will be smoothed an optimal piecewise smooth approximation of the initial, possible noisy image u_0; C has the role of approximating the edges in the image u_0, and u will be smoothed only outside C, i.e. on $\Omega \setminus C$. It allows the segmented objects to have smoothly varying intensities.