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Analytical study of solitons for Lakshmanan–Porsezian–Daniel model with parabolic law nonlinearity

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ABSTRACT

This paper retrieves new solitary wave solutions to the Lakshmanan–Porsezian–Daniel model with parabolic law nonlinearity. The modified trail equation method is used to get dark soliton, rational function and periodic solutions. We also use improved (G'/G)-expansion method to obtain exact and traveling wave solutions.

1. Introduction

Optical solitons have been the subjects of pervasive research in nonlinear optics due to their dormant applications in telecommunication and ultra fast signal processing systems [1–11]. The main nonlinear equation that describes the dynamics of soliton pulses in such media is the nonlinear Schrödinger (NLS) equation that includes the group velocity dispersion (GVD) and self-phase modulation (SPM). This completely integrable equation admits two distinct types of localized solutions, bright and dark soliton solutions [18–40]. One of the famous NLSE is Lakshmanan–Porsezian–Daniel (LPD) model, first observed in 1988, studied in a variety of context including fiber optics. Hubert et al. [12] obtained bright and dark-singular combo soliton for (LPD) model with the help of modified extended direct algebraic method under Kerr law and power law. Jawad et al. [13] used Tanh function method to obtain dark soliton for (LPD) model. They used Kerr law, quadratic law and cubic-quintic nonlinearities. Lie symmetry method with Kerr law and power law was used by Bansal et al. [14] to obtain singular soliton solutions. Biswas et al. [15] used three forms of nonlinearities Kerr law, parabolic law and anti-cubic law to get dark and singular soliton for (LPD) model with the aid of modified simple equation method. Jacobi's elliptic function method and $exp(-phi(\eta))$-expansion method were used by Biswas et al. [16] to get dark and singular soliton for (LPD) model under Kerr law. In [17] Guzman et al. obtained bright, dark and singular soliton for (LPD) model with the help of method of undetermined coefficients under Kerr law and power law. The purpose of this paper is to obtain dark soliton solutions for (LPD) model by using modified trail equation method under parabolic law nonlinearity. We also intend to find the traveling wave solutions, dark and singular soliton solutions with the help of improved $\left(\frac{G'}{G}\right)$-expansion method with parabolic nonlinearity.

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2. Mathematical model

The dimensionless form of LPD model with higher order dimension is given as [12]:

$$iq_t + aq_{xx} + bq_{xt} + cF(|q|^2)q \quad = \sigma q_{xxxx} + \alpha(q_x)^2 q * + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta|q|^4 q = 0 \tag{1}$$

where $q(x,t)$ represents the complex valued wave function. The term q_t represents the temporal evolution of the nonlinear wave, the coefficient a represents GVD, coefficient b represents STD and F is the real-valued algebraic function which is the source of nonlinearity. Fourth order dispersion is represented by σ and two photon absorption by δ. We start with the assumption:

$$q(x,t) = p(\xi)e^{i\phi(x,t)} \tag{2}$$

where $p(\xi)$ represents the shape of wave profile, and the phase constant $\phi(x,t)$ is defined as

$$\begin{aligned} \xi &= x - \upsilon t \\ \phi(x,t) &= -\kappa x + \omega t + \theta \end{aligned} \tag{3}$$

with κ represents the soliton frequency, ω is wave number and θ is the phase constant. By using Eqs. (2) and (3) into Eq. (1), and separating into real and imaginary parts, the real part is

$$\begin{aligned} &\sigma p'''' - (a + 6\sigma\kappa^2)p'' - b\upsilon p'' - (b\kappa\omega - \omega - a\kappa^2 - \sigma\kappa^4)p \\ &-(\alpha + \gamma + \lambda - \beta)\kappa^2 p^3 + \delta p^5 - cF(p^2)p + (\alpha + \beta)pp'^2 + (\lambda + \gamma)p^2 p'' = 0 \end{aligned} \tag{4}$$

and the imaginary part is

$$4\sigma\kappa p'''' + 2(\alpha + \gamma - \lambda)\kappa p^2 p' - (2\alpha\kappa + 4\kappa^3\sigma - b\omega)p' + (-1 + b\kappa)\upsilon p' = 0 \tag{5}$$

By putting coefficient of linear independent functions to zero we get,

$$\sigma = 0, \quad \lambda = \alpha + \gamma \tag{6}$$

and soliton speed

$$\upsilon = \frac{(b\omega - 2\alpha\kappa)}{(1 - b\kappa)} \tag{7}$$

where $1 - b\kappa \neq 0$. After using Eq. (6) into Eq. (4), we get,

$$\begin{aligned} &-ap'' - b\upsilon p'' - (b\kappa\omega - \omega - a\kappa^2)p - (2\lambda - \beta)\kappa^2 p^3 \\ &+\delta p^5 - cF(p^2)p + (\alpha + \beta)pp'^2 + (\lambda + \gamma)p^2 \quad p'' = 0 \end{aligned} \tag{8}$$

In the next section Eq. (8) will be studied with parabolic law nonlinearity,

3. Parabolic law

For parabolic law [15,25], we consider

$$F(p) = p + \nu p^2 \tag{9}$$

By using Eq. (9) into Eq. (8), we get,

$$\begin{aligned} &-ap'' - b\upsilon p'' - (b\kappa\omega - \omega - a\kappa^2)p - (2\lambda - \beta)\kappa^2 p^3 \\ &+\delta p^5 - c(p^3 + \nu p^5)p + (\alpha + \beta)pp'^2 + (\lambda + \gamma)p^2 p'' = 0 \end{aligned} \tag{10}$$

In the next two subsections, Eq. (10) will be used to construct dark, singular and rational function soliton solutions with the help of modified trial equation method and improved $\left(\frac{G'}{G}\right)$-expansion method:

3.1. Modified trial equation method

First of all, we use modified trial equation method [41] to get solitary wave solutions by considering

$$p' = \frac{R(p)}{T(p)} = \frac{\sum_{i=0}^{n} a_i p^i}{\sum_{j=0}^{l} b_j p^j} \tag{11}$$

$$p'' = \frac{R(p)(R'(p)T(p) - R(p)T'(p))}{T^3(p)} \tag{12}$$