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Chirped dispersive bright and singular optical solitons with Schrödinger–Hirota equation

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ABSTRACT

This paper retrieves bright and singular dispersive optical soliton solutions in optical fibers. The governing model is Schrödinger–Hirota equation. The existence criteria for these solitons are also enumerated.

1. Introduction

The dynamics of optical soliton propagation through a variety of waveguides, such as optical fibers, PCF, optical metamaterials, optical couplers, magneto-optic waveguides, is rich and phenomenal. This rich dynamics is administered mathematically for different models using a staggering amount of analytical tools [1–10]. A few of these visible tools are inverse scattering transform, Bäcklund transform, Adomian decomposition scheme, Hirota calculus, semi-inverse variational principle, method of undetermined coefficients, Kudryashov's method, trial equation method, modified simple equation

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scheme, Lie symmetry analysis, mapping methods and several others. These powerful mechanisms can display a plethora of solutions to nonlinear evolution equations (NLEEs), in addition to soliton solutions that are applicable to the photonics area. A few of these additional solutions are shock waves, snoidal and cnoidal waves, periodic-singular waves, plane waves and many more.

This paper will address one such NLEE that is studied in the context of dispersive optical solitons. This is the Schrödinger's equation (SHE) which is considered with the effect of spatio-temporal dispersion (STD) in addition to the usual group velocity dispersion (GVD). A couple of perturbation terms are included to keep the model closer to reality. These stem from inter-modal dispersion and nonlinear dispersion. The traveling wave hypothesis will be applied to reduce the corresponding NLEE to a couple of ordinary differential equations (ODEs). These ODEs will be subsequently analyzed to retrieve chirped dispersive bright and singular optical solitons. After a quick introduction to the model, the details are explored in the rest of the paper.

1.1. Governing model

The perturbed SHE with GVD and STD as studied in [1] is given by:

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q + i\sigma|q|^2 q_x = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x + i\nu\left(|q|^2\right)_x q, \tag{1}$$

where a is the GVD, b represents the coefficient of STD, σ is the nonlinear dispersion, α is the inter-modal dispersion, λ represents the coefficient of self-steepening for short pulses and ν is the higher-order dispersion coefficient.

2. Mathematical analysis

As we interested in solutions with nonlinear chirp of Eq. (1), we start from the representation of the complex field $q(x, t)$ in the form [3–10]

$$q(x,t) = \rho(\xi)e^{i(\chi(\xi)-\Omega t)}, \tag{2}$$

with ρ and χ real functions of $\xi(x, t) = x - ut$. u is the group velocity and Ω the frequency of the wave oscillation. Substituting Eq. (2) into Eq. (1), and collecting the imaginary and real parts yields

$$a\left(\chi_{\xi\xi}\rho + 2\,\chi_{\xi}\rho_{\xi}\right) - \rho_{\xi}u + \sigma\,\rho^2\rho_{\xi} + b\left(-\chi_{\xi}\rho_{\xi}u + \left(-\chi_{\xi}u - \Omega\right)\rho_{\xi} - \chi_{\xi\xi}u\rho\right) - \alpha\,\rho_{\xi} - 3\,\lambda\,\rho^2\rho_{\xi} - 2\,\nu\,\rho^2\rho_{\xi} = 0, \tag{3}$$

$$-a{\chi_{\xi}}^2\rho - \sigma\,\rho^3\chi_{\xi} + \alpha\,\chi_{\xi}\rho + \left(\chi_{\xi}u + \Omega\right)\rho + \lambda\,\rho^3\chi_{\xi} + b\chi_{\xi}\left(\chi_{\xi}u + \Omega\right)\rho + c\rho^3 - b\rho_{\xi\xi}u + a\rho_{\xi\xi} = 0. \tag{4}$$

Integrating Eq. (3) after multiplying it by ρ leads us to obtain,

$$\chi_{\xi} = \frac{(2u + \alpha + b\Omega)}{2(a - bu)} + \frac{(\sigma - 3\lambda - 2\nu)}{4(a - bu)}\rho^2 + \frac{A}{(a - bu)\rho^2}, \tag{5}$$

where A is an integration constant.

The corresponding chirp can be defined by $\delta\omega = -\frac{\partial}{\partial x}[\chi(\xi) - \Omega t] = -\chi_{\xi}$ can be written as

$$\delta\omega(x,t) = -\frac{(2u + \alpha + b\Omega)}{2(a - bu)} + \frac{(3\lambda + 2\nu - \sigma)}{4(a - bu)}\rho^2 - \frac{A}{(a - bu)\rho^2}. \tag{6}$$

Substituting Eq. (5) into Eq. (4) yields

$$m_5\rho^5 + m_3\rho^3 + m_1\rho + (a - bu)\rho_{\xi\xi} - \frac{Au}{(a - bu)\rho} - \frac{A^2}{(a - bu)\rho^3} = 0, \tag{7}$$

where

$$m_5 = -\frac{(-\sigma + 3\,\lambda + 2\,\nu)(-5\,\sigma + 7\,\lambda + 2\,\nu)}{16(a - bu)},$$

$$m_3 = \frac{4\,ca - 2\,\alpha\,\sigma + 2\,\alpha\,\lambda - 4\,cbu + 7\,\lambda\,u - 5\,u\sigma + 2\,u\nu - 2\,b\Omega\,\sigma + 2\,b\Omega\,\lambda}{4(a - bu)},$$

$$m_1 = \frac{4\,\Omega\,a - 6\,A\sigma + 4\,A\nu + b^2\Omega^2 + \alpha^2 + 2\,u\alpha + 10\,A\lambda - 2\,bu\Omega + 2\,\alpha\,b\Omega}{4(a - bu)}.$$

Multiplying Eq. (7) by ρ_{ξ}, integrating with respect to ξ gives,

$$\frac{m_5}{6}\rho^6 + \frac{m_3}{4}\rho^4 + \frac{m_1}{2}\rho^2 + \frac{(a - bu)}{2}\rho_{\xi}^2 - \frac{Au}{(a - bu)}\ln(\rho) + \frac{A^2}{2(a - bu)\rho^2} + B = 0, \tag{8}$$