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Sub-pico-second chirped optical solitons in mono-mode fibers with Kaup–Newell equation by extended trial function method

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ABSTRACT

This paper employs extended trial function method to retrieve sub-pico-second optical soliton solutions to Kaup–Newell's equation that is one of the forms of derivative nonlinear Schrödinger's equation. Bright and singular soliton solutions are revealed with this algorithm.

1. Introduction

One of the several models that is studied to demonstrate soliton molecule propagation through an optical fiber is the derivative nonlinear Schrödinger's equation (DNLSE). There are several forms of DNLSE that are known till today. They are Gerdjikov–Ivanov equation, Chen–Lee–Liu equation and Kaup–Newell equation (KNE). These are alternatively designated as DNLSE-I, DNLSE-II and DNLSE-III equations. The first two models have been extensively studied in the field of nonlinear optics. This paper will study the KNE by the aid of extended trial function scheme in the context of sub-pico-second pulse propagation through an optical fiber. There is a plethora of mathematical schemes that are available today to address these variety of nonlinear evolution equations [1–15]. It is well known that propagation of Alfven waves in plasmas is also modeled by KNE [2]. After a quick introduction to the model, this paper details the derivation of chirped soliton solutions to KNE.

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1.1. Governing model

The dimensionless form of KNE that is going to be studied in this paper is given by [2,3]

$$q_t + iaq_{xx} + b\left(|q|^2 q\right)_x = 0. \tag{1}$$

Here, $q(x, t)$ is a complex-valued function that represents the wave profile. The coefficient of a is the group velocity dispersion and the coefficient of b is the nonlinearity. Soliton solutions are the outcome of a delicate balance that exist between dispersion and nonlinearity.

1.2. Mathematical analysis

In order to investigate solutions with nonlinear chirp of Eq. (1), the following representation of the complex field $q(x, t)$ is adopted:

$$q(x,t) = \rho(\eta)\, e^{i[\chi(\eta)-\omega t]}, \tag{2}$$

where $\eta = x - vt$, $\rho(\eta)$ is the amplitude function, and $\chi(\eta)$ is the phase function. Also, v is the wave velocity, and ω is the frequency of the wave oscillation.

Inserting (2) into (1) and separating into real and imaginary parts yields a pair of relations. Real part gives

$$v\rho' - 3b\rho^2\rho' + 2a\rho'\chi' + a\rho\chi'' = 0, \tag{3}$$

while imaginary part implies

$$\omega\rho + v\rho\chi' - b\rho^3\chi' + a\rho\left(\chi'\right)^2 - a\rho'' = 0, \tag{4}$$

where $\rho' = d\rho/d\eta$, $\rho'' = d^2\rho/d\eta^2$, $\chi' = d\chi/d\eta$ and $\chi'' = d^2\chi/d\eta^2$. Multiplying both sides of (3) by ρ and integrating leads to

$$\chi' = \frac{3b\rho^2}{4a} - \frac{v}{2a} - \frac{A}{a\rho^2}, \tag{5}$$

where A is an integration constant. The corresponding chirp described by

$$\delta\omega = -\frac{\partial}{\partial x}\left[\chi(\eta) - \omega t\right] = -\chi'(\eta), \tag{6}$$

can be written as

$$\delta\omega(x,t) = \frac{A}{a\rho^2} + \frac{v}{2a} - \frac{3b\rho^2}{4a}. \tag{7}$$

Substituting (5) into (4) yields

$$a\rho'' - \frac{A^2}{a\rho^3} + \left(\frac{2bA + v^2 - 4a\omega}{4a}\right)\rho - \frac{bv}{2a}\rho^3 + \frac{3b^2}{16a}\rho^5 = 0. \tag{8}$$

Next balancing the terms ρ'' and ρ^5 gives

$$N = \frac{1}{2}. \tag{9}$$

To obtain a closed form analytic solution, it is employed a transformation formula

$$\rho(x,t) = F^{1/2}(x,t), \tag{10}$$

that will carry (8) into

$$4a^2\left(F'^2 - 2FF''\right) + 16A^2 - 4\left(2bA + v^2 - 4a\omega\right)F^2 + 8bvF^3 - 3b^2F^4 = 0. \tag{11}$$

2. Extended trial function scheme

To start off with extended trial function scheme [4,5,8,9], the following assumption for the solution structure of (11) is taken up:

$$F = \sum_{i=0}^{\varsigma} \tau_i \Phi^i, \tag{12}$$

where

$$(\Phi')^2 = \Lambda(\Phi) = \frac{\Psi(\Phi)}{\Upsilon(\Phi)} = \frac{\mu_\sigma\Phi^\sigma + \cdots + \mu_1\Phi + \mu_0}{\chi_\varrho\Phi^\varrho + \cdots + \chi_1\Phi + \chi_0}. \tag{13}$$