ROBUST FUZZY FAULT DETECTION FOR CONTINUOUS-TIME NONLINEAR DYNAMIC SYSTEMS

Magdy G. El-ghatwary * **Steven X. Ding** ** **Zhiwei Gao** ***

* *Institute for Automatic Control and Complex Systems, University of Duisburg-Essen, Germany (ghatwary@uni-duisburg.de)*
** *Institute for Automatic Control and Complex Systems, University of Duisburg-Essen, Germany*
*** *Department of Automation, Tianjin University, Tianjin 300072, P. R. China (e-mail: zhiwegac@public.tpt.tj.cn)*

Abstract: : A new fault detection scheme for continuous-time nonlinear dynamic system is studied. The nonlinear system is modelled by Takagi-Sugeno fuzzy model. A fuzzy observer-based approach is presented to detect the fault occurred in the dynamic system. The generated residual signal is robust with respect to undesirable effects of unknown inputs and modelling errors but sensitive to faults. By applying H_∞ optimization techniques, a sufficient condition to solvability of the formulated problem is established in terms of Linear Matrix Inequalities. The algorithm for residual evaluation function and threshold computation are also given. Finally, an example is given to illustrate the effectiveness of the proposed design techniques.

Keywords: TS Fuzzy model, fault detection, nonlinear dynamic systems, robust fault detection, fault evaluation

1. INTRODUCTION

Model-based fault detection techniques have been successfully applied to linear systems, see (Patton *et al.*, 1989), (Frank, 1990), (Iserman, 1997), (Patton and Chen, 1997), (Frank and Ding, 1994), (Rambeaux *et al.*, 1999) and (Wang *et al.*, 2003), and nonlinear system (Ding *et al.*, 1999). The main idea is to generate a residual signal that reflects inconsistencies between nominal and faulty system operations. Most studies on nonlinear systems fault detection and isolation (FDI) have been made in two steps. Firstly, the model is linearized at an operating point. Secondly, robust technique is applied to generate residual signal (Chen and Patton, 1996) and (Gertler, 1993). This strategy works well when the linearisation does not cause a large mismatch between linear and nonlinear models and system operates close to the operating point. Therefore, such techniques have limited robustness when considering big plant changes. In the past two decades, many researchers have studied a class of nonlinear systems described by a TS fuzzy model (Takagi and Sugeno, 1985). In this model, a nonlinear dynamic system is described by a number of locally-linearized models. The overall model of system is obtained by blending these linear models through nonlinear membership functions. In other words, TS fuzzy model is essentially a multi-model approach in which simple sub-models are combined to represent the global behavior of the system. This fuzzy model has been shown to be able to approximate a large class of nonlinear systems. FDI based on fuzzy logic has been suggested in (Patton *et al.*, 1998), and the stability as well as eigen-value constraint conditions for fuzzy observer design are also presented and solved in Linear Matrix Inequality. In (Alcorta and Sauceda, 2003), fuzzy observer is designed based on stability condition. In both cases, the model uncertainty and disturbance are not taken into account during fuzzy observer design. In (Gao *et al.*, 2005), a TS fuzzy observer was designed for systems with measurement noises. In (Simani and Patton, 2002) and (Baranyi *et al.*, 2003) using only input/output measurement data to obtain TS fuzzy model. The main objective of this contribution is to study fault detection for continuous-time nonlinear dynamic system. The fault detection system consists of a residual generator and a residual evaluation stage including evaluation function and a threshold. The nonlinear system is represented by TS fuzzy model. The residual generation approach attempts to optimize two contradictory objectives: disturbance attenuation and fault sensitivity. The fault generator is designed so that the L_2 gain from disturbances to residual and L_2 gain from fault to residual satisfy the prespecified constraint of H_∞ norm upper bound. This algorithm is represented in Linear Matrix Inequalities (LMI). The algorithm for residual evaluation function and computation of the threshold using LMI-technique are given. This paper is organized as follows: In Section 2, TS fuzzy model for nonlinear dynamic system is shown. In Section 3, the fault detection design procedure is derived and the design algorithm for the fault detection design is given. The validity of this approach is demonstrated in Section 4 by an example. Finally in Section 5, the conclusion is drawn.

2. TS FUZZY MODEL CONSTRUCTION

The TS fuzzy model with fault and disturbance is described by the following fuzzy IF-THEN rules:
Rule i: IF Z_i is M_{i1} and .... and Z_p is M_{ip} THEN

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + B_{1i}u(t) + B_{2i}w(t) + B_{3i}f(t)\\ y(t) &= C_i x(t) + D_{1i}u(t) + D_{2i}w(t) + D_{3i}f(t)\end{aligned} \quad (1)$$

where $i = 1, ..., r, r$ is the number of If-Then fuzzy rules; $M_{ij}(j = 1, 2, ..., p)$ are fuzzy sets; $Z = [Z_1, ..., Z_p]$ are premise variables, $x(t) \in \mathcal{R}^n$ is state vector; $u(t) \in \mathcal{R}^p$ and $y(t) \in \mathcal{R}^q$ are the input and measure output vectors respectively; $w(t) \in \mathcal{R}^m$ is the disturbance; $f(t) \in \mathcal{R}^l$ is the fault to be detected. $A_i, B_{1i}, B_{2i}, B_{3i}, C_i, D_{1i}, D_{2i}, D_{3i}$ are known matrices with appropriate dimension. The defuzzified output of TS fuzzy system (1) is represented as:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} \mu_i [A_i x(t) + B_{1i}u(t) + B_{2i}w(t) + B_{3i}f(t)]\\ y(t) &= \sum_{i=1}^{r} \mu_i [C_i x(t) + D_{1i}u(t) + D_{2i}w(t) + D_{3i}f(t)]\end{aligned} \quad (2)$$

where $\mu_i(t) = \frac{h_i(Z(t))}{\sum_{i=1}^{r} h_i(Z(t))}$ with $h_i(Z(t)) = \prod_{j=1}^{p} M_{ij}(Z_j(t))$ and $M_{ij}(Z_j(t))$ is the grade of membership $Z_j(t)$ in M_{ij}. Notice that $h_i(Z(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(Z(t)) > 0$ for $i = 1, 2, ..., r$. Therefore $\mu_i(Z(t)) \geq 0$ and $\sum_{i=1}^{r} \mu_i(Z(t)) = 1$. For the sake of simplifying notation $\mu_i(Z(t))$ is replaced by μ_i. The system is assumed to work well in the absence of the fault.

3. RESIDUAL GENERATION

The first step to achieve a successful FD is to generate residual signal which is decoupled from the input signal $u(t)$. In this case, we consider so-called TS fuzzy observer which is described as follows:
Rule i: IF Z_i is M_{i1} and .... and Z_p is M_{ip} THEN

$$\begin{aligned}\dot{\hat{x}}(t) &= A_i \hat{x}(t) + B_1 u(t) + L_i[y(t) - \hat{y}(t)]\\ \hat{y}(t) &= C_i \hat{x}(t) + D_{1i}u(t), R(t) = y(t) - \hat{y}(t)\end{aligned} \quad (3)$$

where L_i is the observer gain matrix for sub-model i and R is residual signal.The fuzzy observer based residual generator is inferred as the weighted sum

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{r} \mu_i [A_i \hat{x}(t) + B_{1i}u(t) + L_i(y(t) - \hat{y}(t))]\\ \hat{y}(t) &= \sum_{i=1}^{r} \mu_i [C_i x(t) + D_{1i}u(t)], R(t) = y(t) - \hat{y}(t)\end{aligned} \quad (4)$$

where μ_i is the same weight function used in TS model (2). To analyze the convergence of the observer, the state error vector $\varepsilon(t) = x(t) - \hat{x}(t)$ is given by the following differential equation.

$$\begin{aligned}\dot{\varepsilon}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j [(A_i - L_i C_j)\varepsilon(t)\\ &+(B_{2i} - L_i D_{2j})w(t) + (B_{3i} - L_i D_{3j})f(t)]\\ R(t) &= \sum_{i=1}^{r} \mu_i [C_i \varepsilon(t) + D_{2i}w(t) + D_{3i}f(t)]\end{aligned} \quad (5)$$

In FDI problem, the residual is expected to be insensitive to unknown disturbance and modelling error, whilst sensitive to fault.

3.1 Problem Formulation

Note that the dynamic of residual signal depends on $f(t)$ and $w(t)$. Thus, the problem of designing TS fuzzy fault detection observer, can be described as designing the observer gain matrix L_i such that the following conditions are simultaneously filled:
(i) $A_i - L_i C_j$ is asymptotically stable for all subsystems A_i.