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Optical solitons for Lakshmanan–Porsezian–Daniel model with dual-dispersion by trial equation method

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ABSTRACT

The trial equation method is applied to obtain soliton solutions to Lakshmanan–Porsezian–Daniel model in optical fibers, PCF and metamaterials. This integration procedure is implemented into the model with three forms of nonlinearity. Bright, dark and singular soliton solutions are recovered with conditions that guarantee their existence.

1. Introduction

Optical soliton propagation across inter-continental distances are governed by a variety of models. They represent the engineering dynamics of such pulse propagation in various circumstances. The most visible model is the familiar nonlinear Schrödinger's equation (NLSE) that has been studied for decades all across the globe. Besides NLSE, there are several models that describe this dynamics fruitfully both in polarization-preserving fibers as well as birefringent fibers and DWDM networks. One such model is the Lakshmanan–Porsezian–Daniel (LPD) model that has been extensively studied [1–10]. This equation first appeared in 1988 and became popular ever since [9]. This paper will cover LPD model by trial equation method that is a fairly popular mathematical scheme to address this form of nonlinear evolution equation. There are three types of nonlinearity that will be addressed and the model will be considered with two forms of dispersion. In addition to the group velocity dispersion (GVD), the spatio-temporal dispersion (STD) will be taken into account. The details are now explored in the remainder of the paper after a quick introduction to the model.

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1.1. Governing model

The LPD model, with higher order dispersion, considered in this work is given by

$$iq_t + aq_{xx} + bq_{xt} + cF(|q|^2)q = \sigma q_{xxxx} + \alpha(q_x)^2 q^* + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta|q|^4 q. \tag{1}$$

In (1), $q(x, t)$ is the complex-valued dependent variable and it represents the wave profile with two independent variables being x and t that represents spatial and temporal components respectively. The first term on the left side represents the temporal evolution of the nonlinear wave, while the coefficient a is GVD and b is the coefficient of STD. The functional F which is the source of fiber nonlinearity is a real-valued algebraic function. The complex-valued function $F(|q|^2)q : C \to C$ must be smooth. Treating the complex plane C as a two-dimensional linear space R^2, the function $F(|q|^2)q$ is k-times continuously differentiable and therefore

$$F(|q|^2)q \in \bigcup_{m,n=1}^{\infty} C^k((-n, n) \times (-m, m); R^2).$$

On the right hand side of (1), σ is the coefficient of fourth order dispersion and δ corresponds to two-photon absorption. The remaining perturbation terms with nonlinear forms of dispersion are indicated by the coefficients of α, β, γ and λ.

2. Quick recapitulation of trial equation method

In this section, the essential steps of the trial equation method are enumerated [11]:

(2) *Step-1*: Suppose a nonlinear evolution equation of the form

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{2}$$

can be transformed to an ordinary differential equation

$$Q(U, U', U'', U''', \ldots) = 0 \tag{3}$$

from the travelling wave hypothesis $u(x, t) = U(\xi)$, $\xi = x - vt$, where $U = U(\xi)$ is the unknown function, Q is a polynomial in variable U and its derivatives. If all terms contain derivatives, then Eq. (3) can be integrated with the integration constant taken to be zero, without any loss of generality.

(3) *Step-2*: Choose the trial equation

$$(U')^2 = F(U) = \sum_{l=0}^{N} a_l U^l \tag{4}$$

where a_l, $(l = 0, a, \ldots, N)$ are constants that needs to be determined. Substituting Eq. (4) and other derivative terms such as U'' or U''' and so on into Eq. (3) gives a polynomial $G(U)$ of U. From the balancing principle we can locate the value of N. Setting the coefficients of $G(U)$ to zero, we arrive at a system of algebraic equations. Solving this system, we can obtain v and values of $a_0, a_1, \ldots, a_N$.

(4) *Step-3*: Rewrite Eq. (4) as

$$\pm(\xi - \xi_0) = \int \frac{dU}{\sqrt{F(U)}} \tag{5}$$

From the discriminant of the polynomial, we classify the roots of $F(U)$, and the evaluate the integral Eq. (5). This enables the recovery of an exact solution to Eq. (2).

3. Soliton solutions

In order to solve Eq. (1) by the trial equation method, we introduce the following transformation

$$q(x, t) = U(\xi)e^{i\phi(x,t)}, \quad q^*(x, t) = U(\xi)e^{-i\phi(x,t)} \tag{6}$$

where $U(\xi)$ represents the shape of the pulse, $\xi = x - vt$ and $\phi(x, t) = -\kappa x + \omega t + \theta$. The function $\phi(x, t)$ is the phase component of the soliton, κ is the soliton frequency, while ω is the wave number, θ is the phase constant and v is the velocity of the soliton.

Substituting Eq. (6) into Eq. (1) and then breaking into real and imaginary parts yields a pair of relations. The real part gives

$$\sigma U'''' - (6\kappa^2\sigma - bv + a)U'' - (b\kappa\omega - \kappa^4\sigma - a\kappa^2 - \omega)U - \kappa^2(\lambda - \beta + \gamma + \alpha)U^3$$

$$+\delta U^5 - cF(U^2)U + (\alpha + \beta)U(U')^2 + (\lambda + \gamma)U^2 U'' = 0 \tag{7}$$