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Dispersive optical solitons with differential group delay and parabolic law nonlinearity by extended trial function method

Anjan Biswas[a,b,c], Mehmet Ekici[d,*], Abdullah Sonmezoglu[d], M.M. Babatin[b]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[b] Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh 13318, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Science and Arts, Bozok University, 66100 Yozgat, Turkey

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ABSTRACT

This paper obtains bright and singular dispersive optical solitons for birefringent fibers in a parabolic law medium. The extended trial function algorithm was implemented to extract these soliton solutions. In addition, cnoidal and snoidal wave solutions are also recovered for the model.

1. Introduction

Pulse splitting in optical fibers is a growing concern in soliton dynamics for optical fibers and PCF. This occurs due to non-uniformity in fiber diameter and is a manufacturing defect. This leads to pulse splitting and hence the effect of differential group delay (DGD) comes up. The cumulative effect of DGD is known as birefringence. This paper addresses the problem for dispersive solitons that is studied in a parabolic law medium and is governed by Schrödinger–Hirota equation (SHE). There are several perturbation terms that are included in the model that arises from various factors in soliton propagation dynamics. A wide variety of proposed mathematical analysis exists to address soliton dynamics in optical fibers and other nonlinear evolution equations [1–10]. This paper will adopt the extended trial function method to analyze the coupled SHE with perturbation terms and parabolic law nonlinearity. Bright and singular soliton solutions emerge from this integration scheme. These soliton solutions exist with certain parameter restrictions that are also presented. After a quick introduction to the model, the detailed derivations are given in the rest of the paper.

1.1. Governing model

In the presence of Hamiltonian type perturbations, the governing equation for the propagation of solitons through birefringent fibers with parabolic law nonlinearity is given by the following SHE [9]:

* Corresponding author.
E-mail address: ekici-m@hotmail.com (M. Ekici).

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$$iq_t + a_1 q_{xx} + (b_1 |q|^2 + c_1 |r|^2)q + (d_1 |q|^4 + e_1 |q|^2|r|^2 + f_1 |r|^4)q + i\lambda_1 q_x + i\nu_1(|q|^2 q)_x + i\xi_1(|q|^2)_x q + i\theta_1 |q|^2 q_x + i\gamma_1 q_{xxx} = 0, \quad (1)$$

$$ir_t + a_2 r_{xx} + (b_2 |r|^2 + c_2 |q|^2)r + (d_2 |r|^4 + e_2 |q|^2|r|^2 + f_2 |q|^4)r + i\lambda_2 r_x + i\nu_2(|r|^2 r)_x + i\xi_2(|r|^2)_x r + i\theta_2 |r|^2 r_x + i\gamma_2 r_{xxx} = 0. \quad (2)$$

In Eqs. (1) and (2), the unknown functions $q(x, t)$ and $r(x, t)$ are the optical wave profiles for the two components in birefringent fibers; x and t represent the spatial and temporal coordinates, respectively. For $l = 1, 2$, the constant parameters a_l, b_l, c_l, λ_l, ν_l and γ_l are, respectively, group velocity dispersion (GVD), self-phase modulation (SPM), cross-phase modulation (XPM), inter-modal dispersion (IMD), self-steepening and third-order dispersion (3OD) for the two polarized pulses. The terms with d_l, e_l and f_l are associated with the quintic nonlinear terms of the parabolic (cubic–quintic) law nonlinearity [2,8,9]. Finally, ξ_l and θ_l are the nonlinear dispersions.

2. Mathematical analysis

In order to seek optical solitons to the coupled system given by (1) and (2), the starting hypothesis is taken up in the form as indicated below:

$$q(x, t) = P_1[\xi(x, t)]\exp[i\phi_1(x, t)], \quad (3)$$

$$r(x, t) = P_2[\xi(x, t)]\exp[i\phi_2(x, t)], \quad (4)$$

where $P_l(\xi)$ for $l = 1, 2$ are the amplitude components of the two solitons and

$$\xi = x - \mathrm{v}\,t, \quad (5)$$

and the phase components ϕ_l are defined as

$$\phi_l = -\kappa_l x + \omega_l t + \theta_l, \quad (6)$$

for $l = 1, 2$. Here, v is the velocity of the solitons, κ_l are frequencies of the two solitons while ω_l are the soliton wave numbers and θ_l are the phase constants. Substituting (3) and (4) into (1) and (2) and splitting into real and imaginary parts give rise to

$$-(\omega_l - \lambda_l\kappa_l + a_l\kappa_l^2 + \gamma_l\kappa_l^3)P_l + (b_l + \nu_l\kappa_l + \theta_l\kappa_l)P_l^3 + c_l P_l P_{\bar{l}}^2 + d_l P_l^5 + e_l P_l^3 P_{\bar{l}}^2 + f_l P_l P_{\bar{l}}^4 + (a_l + 3\gamma_l\kappa_l)P_l'' = 0, \quad (7)$$

and

$$(\lambda_l - 2a_l\kappa_l - 3\gamma_l\kappa_l^2 - v)P_l' + (3\nu_l + 2\xi_l + \theta_l)P_l^2 P_l' + \gamma_l P_l''' = 0, \quad (8)$$

for $l = 1, 2$ and $\bar{l} = 3 - l$. Utilizing the balancing principle leads to

$$P_{\bar{l}} = P_l, \quad (9)$$

and then integrating Eq. (8) with zero constant of integration, one has

$$-(\omega_l - \lambda_l\kappa_l + a_l\kappa_l^2 + \gamma_l\kappa_l^3)P_l + (b_l + c_l + \nu_l\kappa_l + \theta_l\kappa_l)P_l^3 + (d_l + e_l + f_l)P_l^5 + (a_l + 3\gamma_l\kappa_l)P_l'' = 0, \quad (10)$$

$$(\lambda_l - 2a_l\kappa_l - 3\gamma_l\kappa_l^2 - v)P_l + \left(\frac{3\nu_l + 2\xi_l + \theta_l}{3}\right)P_l^3 + \gamma_l P_l'' = 0. \quad (11)$$

Eqs. (10) and (11) will now be analyzed in two different cases.

Case-1: In this case, from (11), the third term implies

$$\gamma_l = 0, \quad (12)$$

for $l = 1, 2$. This means that optical soliton solutions for the governing coupled system, will exist provided third order dispersion vanishes. Therefore, Eqs. (10) and (11), by virtue of Eq. (12), reduces to

$$-(\omega_l - \lambda_l\kappa_l + a_l\kappa_l^2)P_l + (b_l + c_l + \nu_l\kappa_l + \theta_l\kappa_l)P_l^3 + (d_l + e_l + f_l)P_l^5 + a_l P_l'' = 0, \quad (13)$$

$$(\lambda_l - 2a_l\kappa_l - v)P_l + \left(\frac{3\nu_l + 2\xi_l + \theta_l}{3}\right)P_l^3 = 0, \quad (14)$$

respectively. The remaining linearly independent functions, from Eq. (11), bring about the constraint

$$3\nu_l + 2\xi_l + \theta_l = 0, \quad (15)$$

and the velocity of the soliton

$$v = \lambda_l - 2a_l\kappa_l. \quad (16)$$

Equating the two values of the soliton velocity (16) gives