Optik 168 (2018) 553–562

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

A novel shuffling technique based on fractional chaotic maps

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ARTICLE INFO

Article history:
Received 2 September 2017
Received in revised form 15 March 2018
Accepted 11 April 2018

Keywords:
Image encryption technique
Shuffling technique
Fractional chaotic series
Statistical analysis

ABSTRACT

An image encryption technique based on the fractional logistic map is designed in this work. A novel shuffling technique is established by use of fractional chaotic signals. Then it is used to scramble pixel positions. The results are analyzed in comparison with the classical logistic map. Since the employed fractional chaotic map holds complicated dynamics behavior, the encryption result is highly secure. Moreover, by experimental and statistical analysis, we demonstrate that the encryption performance is better than the results in literature.

1. Introduction

As is well-known, the famous Logistic map reads

$$x(n) = \mu x(n-1)(1 - x(n-1)). \tag{1}$$

Particularly, for $3.57 < \mu \leq 4$, Eq. (1) is in a chaotic status. Since chaos has many nice properties, for instance, sensitivity and random–like behavior. Almost from the very beginning, chaos has been extensively used in various engineering problems. As one of the most active areas, image encryption ((IE), for short) based on the chaotic signals has been considered [3,7,8,10,18–20,22,27–33]. In order to improve the security, much effort has been made against illegal copying and distribution on internet [9,11,33].

Recently, it was presented that chaos exists in fractional maps [23]. So then stability theories as well as the applications [24–26] are considered. See the fractional logistic map [23]

$$x(n+1) = x(0) + \frac{\mu}{\Gamma(\nu)} \sum_{i=0}^{n} \frac{\Gamma(n-i+\nu)}{\Gamma(n-i+1)} x(i)(1 - x(i)) \tag{2}$$

which can readily and successively generate fractional chaotic signals as that in (1). The map becomes complicated due to the embedded and varied fractional order ν. For a particular case, if $\nu = 1$, the fractional map shrinks to the classical one. As a result, the chaotic signals driven by fractional operators, demonstrates more complicated behavior than the one generated by the classical logistic map.

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https://doi.org/10.1016/j.ijleo.2018.04.054
0030-4026/© 2018 Published by Elsevier GmbH.

The rest of the paper is constructed in the following. We recall some preliminaries concerning the discrete fractional calculus (DFC) and chaotic results of discrete fractional maps in Section 2. In Section 3, an IE algorithm is designed which includes a modified shuffling method and fractional chaotic diffusion algorithm. In Section 4, experimental results, statistical analysis and key secure analysis are shown.

2. Preliminaries

2.1. Discrete fractional calculus

Definition 1. [1,3]

Let $x(t) : t \in \mathbb{N}_0 := \{0, 1, 2, \ldots\} \to \mathbb{R}$ and $0 < \nu \leq 1$.
The discrete fractional integral is defined as

$$(\Delta_0^{-\nu} x)(t) = \frac{1}{\Gamma(\nu)} \sum_{s=0}^{t-\nu} (t-s-1)^{(\nu-1)} x(s) \tag{3}$$

for $t \in \mathbb{N}_\nu$.

$$({}_C^0\Delta_t^\nu x)(t) = \frac{1}{\Gamma(1-\nu)} \sum_{s=0}^{t-\nu} (t-s-1)^{(-\nu)} x(s) \tag{4}$$

for $t \in \mathbb{N}_\nu$.

Since the discrete kernel functions $(t-s-1)^{(-\nu)}$ are not equal to one, then we can see both of the discrete fractional integral (3) and the fractional difference (4) are operators with discrete memory effects.

Lemma 2. [1]

The discrete Leibniz integral law holds, i.e.,

$$(\Delta_0^{-\nu}({}_C^0\Delta_t^\nu x))(t) = x(t) - x(0), \quad 0 < \nu \leq 1.$$

Given a difference equation,

$$\Delta x(n) = g(n, x(n)), \tag{5}$$

in order to introduce the memory effect and better depict the long interaction of nonlinear systems, we present a fractional difference equation as follows

$${}_C^0\Delta_t^\nu x(t) = g(t+\nu-1, x(t+\nu-1)), \quad t \in \mathbb{N}_{1-\nu}. \tag{6}$$

We take the fractional sum to Eq. (5) and obtain the fractional sum equation in [26]

$$x(t) = x(0) + \frac{1}{\Gamma(\nu)} \sum_{s=1-\nu}^{t-\nu} (t-s-1)^{(\nu-1)} h(s) \tag{7}$$

$$h(s) = g(s+\nu-1, x(s+\nu-1))$$

for $t \in \mathbb{N}_1$, and $0 < \nu \leq 1$. The above treatment is a fractionalization of the classical difference equations [2,4]. Indeed, Eq. (7) can be re-written as (see [23,26])

$$x(n+1) = x(0) + \frac{1}{\Gamma(\nu)} \sum_{j=0}^{n} \frac{\Gamma(n-j+\nu)}{\Gamma(n-j+1)} g(j, x(j)). \tag{8}$$

2.2. Chaos in discrete fractional logistic map

Let us generally revisit some results of the fractional logistic map (2). The dynamic behaviors are numerically discussed for the varied parameters μ and ν. For example, the parameters are set respectively as follows $x(0) = 0.3$ and $\nu = 0.6$; $x(0) = 0.3$ and $\nu = 0.01$; $x(0) = 0.3$ and $\nu = 1$. The bifurcation diagrams are plotted in Fig. 1. For the fractional case, from the positive Lyapunov exponents, we can see that the system is in the chaotic status, see Fig. 2. The concept of the fractional chaotic series or signals is not new. In fact, there are rich results about the chaos of fractional differential equations (FDEs, for short) studied [5,6,12–17]. It is needed to point out that [14] used the transfer function to investigate the fractional system. Then, they also employed the predictor corrector method to give chaotic solutions numerically as well as the chaotic breather and fractal dimension of the fractional Chen system. In addition, the fractional map is another novel fractional model which belongs