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A numerically stable algorithm for scattering from several circular cylinders including metamaterials with different boundary conditions

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ABSTRACT

The analytically obtained algebraic equation systems for the TM/TE-z polarized monochromatic waves scattering from eccentrically layered circular cylinders are ill-conditioned for numerical calculations. Therefore, such ill conditioned systems must be regularized for reliable numerical results. Here, the steps of the regularization of the ill conditioned system obtained for the scattered field from a circular metamaterial cylinder includes three parallel circular cylinders: a dielectric, an impedance and a perfect electric conductor is explained. In the regularization procedure used here each circular boundary brings block(s) correspond to its electromagnetic property and the regularization procedure is done according to this block(s). For this end a system that consists of perfect electric conductor (PEC), impedance, dielectric, and metamaterial cylinders is discussed and thus the steps of a regularization procedure for a general system that has all the type of boundaries in terms of electromagnetics will be explained here. As a result, if a new boundary is included to the system then it is sufficient to locate the blocks related to this boundary suitably.

1. Introduction

Currently, many researches employ analytical methods for scattering from canonical structures, since they help understanding basics and comparing the results of new numerical methods in electromagnetic. Thus, the reliability of the scattering formulation from a combination of circularly cylindrical boundaries is of utmost importance. For this purpose, integral formulation for scattering from such boundaries is mandatory [1]. One can follow how it was applied to cylindrical boundaries with impedance conditions [2], and dielectric boundaries [3]. The key to numerically stable formulation there benefits from the ideas of the organization of the scattering matrix in [4]. Basically, this is called analytical regularization [5]. Its subject is to well-condition the linear algebraic system of the first kind (LAES1- in view $\mathbf{Ax}=\mathbf{b}$) that one may reach via Green's integral identities or separation of variables for the scattering problem under consideration. This system is ill-conditioned and prone to round off errors when truncated during its inversion. That is why [4,5] suggests the ways to represent it as a

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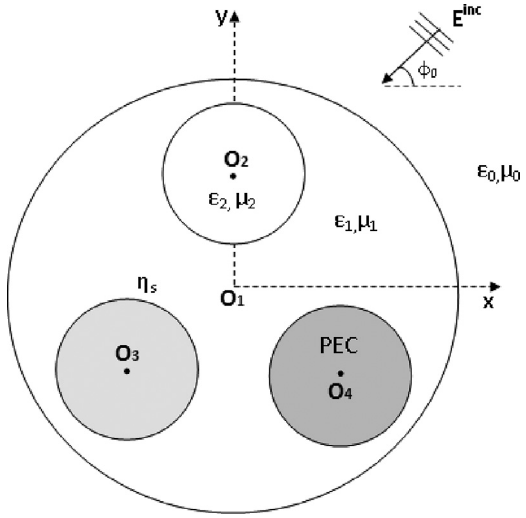

Fig. 1. Geometry of the system with impedance, PEC and dielectric boundaries.

linear algebraic system of the second kind (LAES2- in view **(I + K) = g**) via analytical regularization. Here **A** and **K** are compact operators in space l_2 and **I** is the identity operator. It is well known that LAES2 is immune to round off errors when truncated during its inversion.

Nowadays, studies on metamaterials for scattering become very significant due to the great potential application of them. Metamaterials are artificially constructed materials which have simultaneously negative permittivity and permeability over a particular range of frequency which cannot be easily found in the nature [6–11]. They can be created by using Lorentz and Drude medium models as found in different studies [10–33], in which the permittivity and permeability of such models can be negative both analytically and physically. Lossy and lossless Lorentz and Drude types of metamaterials were described theoretically, used in many experiments for different studies, and manufactured for new devices and applications [34]. Lorentz type metamaterials can be fabricated using wire strip and split ring resonators [10,22]. Furthermore, it is possible to manufacture them on a printed circuit boards by using mixture of conductive spirals [12,14,23]. Besides that, studies on many different forms of Lorentz and Drude types of metamaterials can be found in the literature [34].

This study deals with a scattering problem under TM-z polarized plane wave illumination that provides an example for how several circularly cylindrical boundaries with different boundary conditions can be combined and uses metamaterial parameters for one of the dielectric regions and one of the impedance boundaries. Although the explanation is given for TM-z illumination the TE-z version is very similar to shown here and omitted. The exemplification involves their analytical regularization and one can relate the performed steps to scattering from other configurations of circularly cylindrical boundaries that can involve arbitrary number of inclusions and neighbors. Analytical regularization basically requires a right-hand side invertible operator **R** that **y=R**$^{-1}$**x** is valid between the solutions of the LAES1 and LAES2. Then it may or may not be required (former in this work) to configure a left-hand side invertible operator **L** that **LAR = I + K** is valid with the properties given above in space l_2. [2,3] shows the excellent numerical properties of such an infinite system when truncated in cases when boundary conditions are impedance, penetrable dielectric and [35,36] explain the necessity of transformation to such a system for a metamaterial covered perfectly electrically conductive cylinders. Numerical results given here involve such impressive numerical properties established for the considered scattering mechanism. They involve the condition numbers and ranks of the matrices as well as the success of satisfaction of the boundary conditions in each boundary comparing the results from LAES1 and LAES2.

2. Geometrical structure of the problem and formulation

Geometry of the system consists of circularly cylindrical regions homogenous along $\mathbf{O}_z$ axis as depicted in Fig. 1. As can be observed there, $\varepsilon_{1,}\ \mu_1$ are dielectric permittivity and magnetic permeability of the host region -region 1- which is a metamaterial. The three circular cylindrical boundaries in it are region 2 with constitutive parameters $\varepsilon_{2,}\ \mu_2$ - a non-magnetic dielectric-, region 3 with its surface characterized by surface impedance η_s representing the condition of the boundary of a metamaterial region, and region 4 with a perfectly electrically conductive boundary. TM-z polarized plane wave impinging on the system with an angle φ_0 can be expanded to cylindrical wave functions via the following identity.

$$E_z^{inc}(\rho,\varphi) = e^{-ik\rho\cos(\varphi-\varphi_0)} = \sum_{n=-\infty}^{\infty} i^{-n} J_n(k\rho) e^{in(\varphi-\varphi_0)} \tag{1}$$