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Wave equation for the energy and the momentum of a moving particle

C.Z. Tan

Bowei Geophysics Ltd., Xiangtan National High-Tech Center, 411104 Xiangtan, Province Hunan, PR China

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ABSTRACT

The conserved momentum and the energy of a moving particle obey a wave equation and have the wave feature. The moving particle acts as a point source of the waves. This nature is applied to explain the self-interference of an electron in Young's two-slit interference experiment and the material dispersion. Material dispersion is associated with the rest mass of a photon in a medium.

1. Introduction

Wave-particle duality is normally a fundamental property of quantum mechanics, of which the wave nature is described by a complex wave function [1,2]. The wave function itself is a non-measurable and non-local quantity. According to Born's explanation, the probability of finding a particle at a particular point within a volume is proportional to the square of the wave function. The fictive complex wave function obeys a wave equation and satisfies the Schrödinger equation [2].

A moving particle carries out the momentum and the energy. Within the framework of classical physics, the motion of the particle is governed by Newton's laws, or by the theory of special relativity, with the quantities of the force, mass, momentum, and energy, while propagation of the wave is described by the wave equation. For both particle and the wave modes, the homogeneous wave equation, the energy, and the momentum are generally the functions of time and space coordinates, and are all extensive quantities which satisfy the principle of superposition [2]. It is found that the conserved momentum and the energy of the moving particle obey the wave equation. The wave velocity is equal to the velocity of the particle [3–5]. In contrast with the fictive wave function in quantum mechanics, the energy and the momentum are all measurable properties in classical physics.

The characteristic phenomena of the wave property are diffraction and interference. Historically a two-slit interference experiment are used to demonstrate the wave nature of the photons and the electrons after passing through the slits [2,6]. Although the photon and the electron are different particles, the emergence of the interference patterns are usually explained by the linear superposition of the traveling waves. However, self-interference of one electron has been recorded in the two-slit interference experiment [7]. Under this circumstance it is impossible to have any sort of cooperative interaction between the electrons of the beam. Theoretically the interference pattern represents the spatial distribution of the intensity or energy flow. When the energy of the moving electron satisfies the wave equation, then it is the non-local property. Hence

E-mail address: bowei_geophysics@sina.com

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the energy waves corresponding to the electron are able to pass through two slits and generate the interference pattern. Self-interference of the electron is in fact an experimental confirmation of the energy wave, which will be discussed in detail.

Interaction of light with matter remains a long standing research object. An electromagnetic wave in vacuum of the frequency ν and the wavelength λ carries out an energy $h\nu$, and the *de Broglie* momentum h/λ, where h is Planck's constant. The light wave in vacuum is massless and non-dispersive [2]. When the light wave is incident into a transparent dielectric medium, the phase velocity becomes c/n, which is less than the speed of light c, where n is the index of refraction [6]. Light wave in the medium thus has mass [3–5]. In this case, the momentum, the energy, as well as the moving mass of the corresponding particle are found to be the functions of the index of refraction [3–5]. Hence material dispersion provides reliable data to reveal the nature of the massive photon in the medium, and is a confirmation of wave-particle duality in classical physics.

2. Wave equation for particle mode

When a particle of the rest mass m travels at a speed u, the linear momentum, p, and the total energy, W, of the particle are described by [8,9]

$$p = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}}, \tag{1}$$

$$W = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}}. \tag{2}$$

In the theory of relativity, the linear momentum and the total energy are related by the following invariant equation:

$$W^2 = (cp)^2 + (mc^2)^2. \tag{3}$$

Note that W and p are related to u and are generally the functions of time and space coordinates. Differentiating Eq. (3) with respect to time t, one has

$$\frac{\partial p}{\partial t} = \frac{1}{u}\frac{\partial W}{\partial t}. \tag{4}$$

Suppose the particle moves along the x-axis. The velocity is then defined as

$$u = \pm\frac{dx}{dt}. \tag{5}$$

Combining Eq. (5) with (4) we have

$$\frac{\partial p}{\partial t} = \pm\frac{\partial W}{\partial x}. \tag{6}$$

Partially differentiating Eq. (3) with respect to x yields

$$\frac{\partial W}{\partial x} = u\frac{\partial p}{\partial x}. \tag{7}$$

Substituting Eq. (7) into (6), and Eqs. (4), (6) into (7) leads to the following partial differential equations:

$$\frac{\partial p}{\partial t} \mp u\frac{\partial p}{\partial x} = 0, \tag{8}$$

$$\frac{\partial W}{\partial t} \mp u\frac{\partial W}{\partial x} = 0. \tag{9}$$

Mathematically the total derivatives dp/dt and dW/dt are given by

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + \frac{\partial p}{\partial x}\frac{dx}{dt} = \frac{\partial p}{\partial t} \pm u\frac{\partial p}{\partial x}, \tag{10}$$

$$\frac{dW}{dt} = \frac{\partial W}{\partial t} + \frac{\partial W}{\partial x}\frac{dx}{dt} = \frac{\partial W}{\partial t} \pm u\frac{\partial W}{\partial x}. \tag{11}$$

Comparing Eq. (8) with (10), and (9) with (11), one finds: $dp/dt = 0$, and $dW/dt = 0$. These results indicate that a force, $F = dp/dt$, acting on the particle, and the power, dW/dt, are equal to zero. Hence Eqs. (8) and (9) represent the conversations of the momentum and the energy, respectively. Multiplying the expressions depicted in Eqs. (8) and (9) yields

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - u\frac{\partial}{\partial x}\right)p^2 = 0, \tag{12}$$