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Sub pico-second pulses in mono-mode optical fibers with Kaup–Newell equation by a couple of integration schemes

Anjan Biswas[a,b,c], Yakup Yıldırım[d], Emrullah Yaşar[d], Qin Zhou[e,*], Seithuti P. Moshokoa[c], Milivoj Belic[f]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[b] Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh 13318, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Arts and Sciences, Uludag University, 16059 Bursa, Turkey
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[f] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

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ABSTRACT

This paper examines soliton dynamics of sub pico-second pulses modeled by Kaup–Newell equation. The modified simple equation scheme and trial equation approach retrieves dark, bright as well as singular solitons to the model. The constraint relations guarantee the existence of such solitons.

1. Introduction

The model of sub pico-second pulse propagation was first proposed by David Kaup and Alan Newell several decades ago and hence the model became popularly known as the Kaup–Newell equation (KNE) [1–10]. This KNE is also occasionally referred to as derivative nonlinear Schrödinger's equation (DNLSE). This is one of the three forms of DNLSE, where the other two forms are Gerdjikov–Ivanov equation and Chen–Lee–Liu equation. The last two models have been extensively studied over the past couple of years and several interesting results have been reported. The current paper will focus on KNE that will be addressed by two integration schemes where sub pico-second optical soliton solutions will be retrieved in nonlinear optics arena. It must be noted that KNE models Alfven waves in plasmas as well, and that was studied earlier in 2005 [2]. After a quick review of the two integration schemes, the subtraction of soliton solutions of FNE will be handled in the remaining of the manuscript.

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

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1.1. Governing model

The KNE will be employed as follows

$$q_t + iaq_{xx} + b\left(|q|^2 q\right)_x = 0. \tag{1}$$

In Eq. (1), t and x stands for temporal and spatial components separately and they are independent variables. The solitary wave profile corresponds to $q(x, t)$ which is the dependent variable and a complex-valued function. Also, the first term in (1) indicates the linear temporal evolution of pulses. The nonlinearity coefficient is b and coefficient a means to the group velocity dispersion (GVD). We emphasize that both a and b are considered as real constants. The consequence of a sensitive equilibrium among GVD and the nonlinear term are these sub pico-second pulses. The derivation of soliton solutions is employed the rest of the paper.

2. Quick glance at trial equation method

The fundamental stages of this method are enumerated by following steps:

Step-1: Let's consider a nonlinear evolution equation (NLEE)

$$\Lambda\left(q, q_t, q_x, q_{tt}, q_{xt}, q_{xx}, \ldots\right) = 0. \tag{2}$$

Eq. (2) transforms to ordinary differential equation (ODE)

$$\Delta\left(Q, Q', Q'', Q''', \ldots\right) = 0 \tag{3}$$

by help of the wave variable $q(x, t) = Q(\zeta)$, $\zeta = x - vt$, where $Q = Q(\zeta)$ stands for an dependent function when Δ implies to a polynomial which includes Q with its derivatives.

Step-2: In what follows, we consider the following auxiliary first order ODE

$$\left(Q'\right)^2 = H(Q) = \sum_{i=0}^{M} \delta_i Q^i \tag{4}$$

where $\delta_0, \delta_1, \ldots, \delta_M$ are unknown coefficients and will be later fixed. Plugging Eq. (4) and necessary derivatives terms into Eq. (3), one obtains an expression $\Phi(Q)$. In (4), the positive value integer of M which is the order of the finite series can be achieved by balancing rule. One comes up with a overdetermined system which contains unknown parameters under the condition of setting the coefficients of $\Phi(Q)$ to zero. With help of symbolic computation softwares, the constants of v, δ_0, δ_1, $\ldots$, δ_M are fixed.

Step-3: Reformulate Eq. (4) with an integral representation as

$$\pm(\zeta - \zeta_0) = \int \frac{dQ}{\sqrt{H(Q)}} \tag{5}$$

Based on the classification of the discriminants of the integrand, one evaluates the integral Eq. (5) by categorizing the roots of $H(Q)$. So, analytical solutions for Eq. (2) are recovered.

2.1. Application to KNE

In order to apply the aforementioned method for solving the KNE the solution form will be assumed as follows

$$q(x, t) = Q(\zeta)e^{i\phi(x,t)}, \tag{6}$$

where the parameter ζ is known as wave variable and can be described as

$$\zeta = x - vt \tag{7}$$

while the phase function $\phi(x, t)$ is given by

$$\phi(x, t) = -\kappa x + \omega t + \theta \tag{8}$$

where the parameters κ, ω and θ are soliton frequency, soliton wave number and soliton phase respectively.

Plugging (6) into (1), the real and imaginary components give rise to

$$v = 2a\kappa + 3bP^2 \tag{9}$$

and

$$aP'' + \left(\omega - a\kappa^2\right)P - b\kappa P^3 = 0. \tag{10}$$

respectively.

Case-1: