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Optical solitons with differential group delay and four-wave mixing using two integration procedures

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ABSTRACT

Optical solitons in birefringent fibers with the effect of four-wave mixing having both parabolic and Kerr law nonlinearity are obtained by the modified simple equation scheme and the trial equation approach. Dark, bright as well as singular soliton type solutions are derived. Also, the two integration schemes give rise to additional solutions such as singular periodic solutions.

1. Introduction

The phenomenon of birefringence or double refraction in an optical fiber is inevitable. This comes with the splitting of pulses into two that leads to differential group delay. The cumulative effect of this delay implies birefringence. Most of the mathematical analysis for birefringence are carried out without the effect of four-wave mixing (4WM). However, there are quite a few results that are reported with the inclusion of 4WM effect in birefringent fibers for parabolic law and Kerr law nonlinearity [1–10]. This paper studies the effect of birefringence in optical fibers with 4WM effect in parabolic and Kerr law nonlinear fibers. The trial equation methodology and the modified simple equation approach are applied to retrieve dark, bright with singular soliton type solutions for the model of study. With the inclusion of 4WM effect, phase-matching condition is implemented to permit integrability of the governing model. Not only the effect of spatio-temporal dispersion (STD) but also group velocity dispersion (GVD) is included. The remainder of the paper will yield the particulars of the subtraction of soliton solutions.

2. A quick brush-up of trial equation method

The fundamental stages of this method are enumerated by following steps:

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Step 1: Let's consider a nonlinear evolution equation (NLEE)

$$\Lambda(q, q_t, q_x, q_{tt}, q_{xt}, q_{xx}, \ldots) = 0. \tag{1}$$

Eq. (1) transforms to ordinary differential equation (ODE)

$$\Delta(Q, Q', Q'', Q''', \ldots) = 0 \tag{2}$$

by help of the wave variables $q(x, t) = Q(\zeta)$, $\zeta = x - vt$, where $Q = Q(\zeta)$ stands for an dependent function when Δ implies to a polynomial which includes Q with its derivatives.

Step 2: In what follows, we consider the following auxiliary first order ODE

$$(Q')^2 = H(Q) = \sum_{i=0}^{M} \delta_i Q^i \tag{3}$$

where δ_0,δ_1,. . .,δ_M are unknown coefficients and will be later fixed. We have a polynomial expression in $\Phi(Q)$ because of plugging Eq. (3) and necessary derivatives terms into Eq. (2). In Eq. (3), the positive integer of M which is the order of the finite series can be achieved by balancing rule. One comes up with a overdetermined system which contains unknown parameters under the condition of setting the coefficients of of $\Phi(Q)$ to zero. With help of symbolic computation softwares, the constants of v,δ_0,δ_1,. . ., δ_M are fixed.*Step 3*: Reformulate Eq. (3) with an integral representation as

$$\pm(\zeta - \zeta_0) = \int \frac{dQ}{\sqrt{H(Q)}} \tag{4}$$

Based on the classification of the discriminants of the integrand, one evaluates the integral Eq. (4) by categorizing the roots of $H(Q)$. So, analytical solutions for Eq. (1) are recovered.

2.1. Kerr law

The coupled nonlinear Schrödiner equation (NLSE) throughout Kerr law nonlinearity will be employed as follows

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + (\rho_1 |q|^2 + \gamma_1 |r|^2) q + i\lambda_1 q_x + \beta_1 q + \sigma_1 q^* r^2 = 0, \tag{5}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + (\rho_2 |r|^2 + \gamma_2 |q|^2) r + i\lambda_2 r_x + \beta_2 r + \sigma_2 r^* q^2 = 0. \tag{6}$$

In (5) and (6), t and x stand for temporal and spatial components respectively and they are independent variables. Then, the solitary wave profile for the two components in birefringent fibers corresponds to $r(x, t)$ as well as $q(x, t)$ which demonstrate not only the dependent variables but also complex-valued functions. Also, the first terms in (5) and (6) indicate the linear temporal evolution of pulses through birefringent fibers. When the coefficients b_1,b_2 imply to STD terms, the coefficients a_1,a_2 are GVD terms along the two components. For $j = 1, 2$, γ_j means cross-phase modulation (XPM) whilst ρ_j signifies self-phase modulation (SPM). While β_j are proportional to difference between propagation constants, λ_j is proportional to inverse group velocity difference and σ_j accounts for 4WM.

To construct the exact solutions of Eqs. (5) and (6) by help of above algorithm, the solution form will be assumed as

$$q(x, t) = Q_1(\zeta) e^{i\phi(x,t)}, \tag{7}$$

$$r(x, t) = Q_2(\zeta) e^{i\phi(x,t)}, \tag{8}$$

in which Q_1,Q_2 indicates the amplitude component of the soliton when the parameter ζ is known as wave variable and can be described by

$$\zeta = x - vt \tag{9}$$

where v implies to the speed of the soliton. Finally, $\phi(x, t)$ is phase component and is defined as

$$\phi(x, t) = -\kappa x + \omega t + \theta \tag{10}$$

where the parameters κ, ω and θ mean the soliton frequency, soliton wave number and soliton phase respectively. Plugging (7) and (8) into (5) and (6), the imaginary and real components give rise to

$$v = \frac{-2\kappa a_l + \omega b_l + \lambda_l}{1 - \kappa b_l} \tag{11}$$

and

$$(a_l - v b_l) Q_l'' - (\omega + \kappa^2 a_l - \kappa\omega b_l - \kappa\lambda_l - \beta_l) Q_l + \rho_l Q_l^3 + (\gamma_l + \sigma_l) Q_l Q_{\bar{l}}^2 = 0 \tag{12}$$

respectively. Employing the balancing rule is concluded to

$$Q_{\bar{l}} = Q_l, \tag{13}$$