FUZZY PARITY EQUATION FOR FAULT DETECTION AND IDENTIFICATION OF NONLINEAR SYSTEMS

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Abstract: A fault detection and identification method for nonlinear systems based on fuzzy parity equations is presented. The proposed technique involves approximating first the nonlinear system by a T-S model, and then fully-decoupled parity equations are derived for the local linear models. Residuals generated from the fuzzy parity equations in the T-S model are fused by another T-S model. As the residual generated by the fuzzy parity equation is sensitive to a specific actuator or sensor fault, and insensitive to other faults, system states and disturbance inputs, faults can be detected and/or identified. A simulation example involving a nonlinear airplane model is presented to illustrate the performance of the proposed technique.

Key words: Fault diagnosis ; fault detection and identification; parameter estimation; fuzzy model; nonlinear systems

1. INTRODUCTION

To improve the safety and reliability of engineering systems, fault diagnosis is now an important part of a control system. Parity equation is a general approach to detect faults. Residuals are computed by comparing the model output with actual data. If the system is operating normally, the residuals generated by the parity equation is small, but becomes significant otherwise. Therefore, faults can be detected from these residuals. If the residuals are sensitive to specific faults, then these faults can be isolated and/or identified. Although many techniques have been derived using this approach, (see Chow and Willsky, 1984; Lou *et al.*, 1986; Zhang and Patton, 1993; Jin and Zhang, 1999; Jin and Zhang, 2000), these techniques are derived assuming an accurate mathematical model of the system is available, and that the system is linear. Very few results are available in the literature for detecting or identifying faults of nonlinear systems.

The fault detection and identification of nonlinear systems is a complex problem. If there are sufficient data, models, such as the T-S model (Peter, 1999; Javier, *et al.*, 2000) can be estimated to approximate the nonlinear system, from which suitable fault diagnosis techniques can be devised to detect and/or identify faults. A major advantage of the T-S model is that it consists of linear models. Consequently, fault diagnosis techniques can be derived based on the T-S model by extending those techniques derived for linear systems. As the parity equation approach is manly derived for linear system, a T-S model can be used to fuss the residuals generated from the parity equations of the local linear models. Since the local linear models at the selected operating points are time invariant, these models can be trained offline first, and subsequently the fuzzy parity equation can also be trained off-line. However, the diagnosis of sensor fault using this approach has not been considered by Schram *et al.* (1998).

Fuzzy parity equation is developed following the approach presented by Schram, *et al.*, (1998) in this paper for generating residuals by a T-S model and the fully-decoupled parity equation. First, the nonlinear system is approximated by a T-S model using local linear models estimated at selected operating points. Second, fully-decoupled parity equations are derived for these local models for generating residuals that are sensitive to specific actuator or sensor fault, but insensitive to other faults, system states and disturbance inputs. The fuzzy parity equation is obtained by using a T-S model to fuse these residuals. From the residuals generated by the fuzzy parity equation, faults are detected and/or identify.

In Section 2, the description of nonlinear systems and the T-S model for approximating them is presented. The fuzzy parity equation is described in section 3, and the use of the fuzzy parity equation to detect and /or identify actuator and sensor fault is given in section 4. In section 5, the implementation and the performance of the proposed technique is illustrated by a simulation example involving an aircraft are presented.

2. DESCRIPTION OF NONLINEAR SYSTEMS

T-S fuzzy model is a universal approximator (Wang, 1994) for nonlinear systems. It involves formulating the nonlinear system as a fusion of the local linear models at selected operating points, as described below. Consider the nonlinear system,

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)] \\ \mathbf{y}(t) = \mathbf{h}[\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)] \end{cases} \qquad (1)$$

$\mathbf{x}(k) \in R^n$ is the state, $\mathbf{u}(k) \in R^p$ is the input, $\mathbf{y}(k) \in R^q$ is the sensor output, $\mathbf{w}(t) \in R^r$ is the reference input, $\mathbf{f}$[.] and $\mathbf{h}$[.] are smooth nonlinear functions. Let there be m local linear models selected at the operating points, l, l = 1, 2, .., m, each of which is given by,

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}_l \mathbf{x}(k) + \mathbf{B}_l \mathbf{u}(k) + \mathbf{F}_l \mathbf{w}(k) + ?(k) \\ \mathbf{y}(k) = \mathbf{C}_l \mathbf{x}(k) + \mathbf{G}_l \mathbf{w}(k) + ?(k) \end{cases} \qquad (2)$$

$?(k)$ and $?(k)$ are the white noises, $\mathbf{A}_l$, $\mathbf{B}_l$, $\mathbf{C}_l$, $\mathbf{F}_l$ and $\mathbf{G}_l$ are known matrices with appropriate dimensions, the subscript l denotes the respective operating point. For the local models given by (1), the T-S model is given by a set of m "IF-THEN" fuzzy rules, where the l^{th} rule is given by,

Rule l:

IF $y_1(k)$ is S_{l1}, and $y_2(k)$ is S_{l2}, and…, THEN

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}_l \mathbf{x}(k) + \mathbf{B}_l \mathbf{u}(k) + \mathbf{F}_l \mathbf{w}(k) + ?(k) \\ \mathbf{y}(k) = \mathbf{C}_l \mathbf{x}(k) + \mathbf{G}_l \mathbf{w}(k) + ?(k) \end{cases}$$

where $? = [y_1 \ \ y_2 \ \ \cdots \ \ y_r]^T$ is the premise variable and $\{S_{lj},\ j = 1,2,\cdots,r\}$ the fuzzy set. Let $\{m_{s_{lj}}(y_j(k))\}$ be the membership functions of the fuzzy set. Then the global state and output of the T-S model are:

$$\begin{cases} \mathbf{x}(k+1) = \sum_{l=1}^{m} b_l^*(?(k))\left[\mathbf{A}_l \mathbf{x}(k) + \mathbf{B}_l \mathbf{u}(k) + \mathbf{F}_l \mathbf{w}(k) + ?(K)\right] \\ \mathbf{y}(k) = \sum_{l=1}^{m} b_l^*(?(k))\left[\mathbf{C}_l \mathbf{x}(k) + \mathbf{G}_l \mathbf{w}(k) + ?(K)\right] \end{cases} \qquad (3)$$

where m is the number of local models, the same as the number of fuzzy rules, and $b_l^*(?(k))$ is given by,

$$b_l^*(?(k)) = \frac{1}{\sum_{l=1}^{m} b_l(?(k))} b_l(?(k)) \qquad (4)$$

The degree of fulfillment of the fuzzy rule l, $b_l(?(k))$, is:

$$b_l(?(k)) = \prod_{j=1}^{r} m_{s_{lj}}(y_j(k)), \qquad (5)$$

For a data window with $s > 0$, the measurement equation for the local model given by (2) is (Jin and Zhang, 1999):

$$\mathbf{Y}(k) = \mathbf{H}_{l0}\mathbf{x}(k-s) + \mathbf{H}_{lc}\mathbf{U}(k) + \mathbf{H}_{lw}\mathbf{W}(k) \qquad (6)$$

$\mathbf{U}(k) = [\mathbf{u}^T(k-s)\ldots\mathbf{u}^T(k)]^T$, $\mathbf{W}(k) = [\mathbf{w}^T(k-s)\ldots\mathbf{w}^T(k)]^T$, and $\mathbf{Y}(k) = [\mathbf{y}^T(k-s)\ldots\mathbf{y}^T(k)]^T$ is the sensor output,

$$\mathbf{H}_{l0} = \begin{bmatrix} \mathbf{C}_l \\ \mathbf{C}_l\mathbf{A}_l \\ \vdots \\ \mathbf{C}_l\mathbf{A}_l^{s-1} \\ \mathbf{C}_l\mathbf{A}_l^{s} \end{bmatrix},$$

$$\mathbf{H}_{lc} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & & \cdots & & \mathbf{0} \\ \mathbf{C}_l\mathbf{B}_l & \mathbf{0} & \mathbf{0} & \cdots & & \mathbf{0} \\ \mathbf{C}_l\mathbf{A}_l\mathbf{B}_l & \mathbf{C}_l\mathbf{B}_l & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & & & \\ \mathbf{C}_l\mathbf{A}_l^{s-2}\mathbf{B}_l & \cdots & \cdots & \mathbf{C}_l\mathbf{B}_l & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_l\mathbf{A}_l^{s-1}\mathbf{B}_l & \mathbf{C}_l\mathbf{A}_l^{s-2}\mathbf{B}_l & \cdots & \mathbf{C}_l\mathbf{A}_l\mathbf{B}_l & \mathbf{C}_l\mathbf{B}_l & \mathbf{0} \end{bmatrix},$$

and

$$\mathbf{H}_{lw} = \begin{bmatrix} \mathbf{G}_l & \mathbf{0} & & \cdots & & \mathbf{0} \\ \mathbf{C}_l\mathbf{F}_l & \mathbf{G}_l & \mathbf{0} & \cdots & & \mathbf{0} \\ \mathbf{C}_l\mathbf{A}_l\mathbf{F}_l & \mathbf{C}_l\mathbf{F}_l & \mathbf{G}_l & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & & & \\ \mathbf{C}_l\mathbf{A}_l^{s-2}\mathbf{F}_l & \cdots & \cdots & \mathbf{C}_l\mathbf{F}_l & \mathbf{G}_l & \mathbf{0} \\ \mathbf{C}_l\mathbf{A}_l^{s-1}\mathbf{F}_l & \mathbf{C}_l\mathbf{A}_l^{s-2}\mathbf{F}_l & \cdots & \mathbf{C}_l\mathbf{A}_l\mathbf{F}_l & \mathbf{C}_l\mathbf{F}_l & \mathbf{G}_l \end{bmatrix}$$

3. FUZZY PARITY EQUATION

Let the residual generated by the parity equation at operating point l be $r_l(k)$, l = 1, 2, .., m. The T-S model is given by:

Rule l:

IF $y_1(k)$ is S_{l1}, and $y_2(k)$ is S_{l2}, and…, THEN the residual $r_l(k)$ generated by the parity equation is,

$$r_l(k) = \mathbf{v}_l^T\left[\mathbf{Z}(k) - \mathbf{H}_{lc}\mathbf{U}_c(k)\right] \qquad (7)$$

where $\mathbf{U}_c(k) = [(\mathbf{u}_c(k-s))^t \ldots (\mathbf{u}_c(k))^t]^T$ is the normal actuator input, $\mathbf{Z}(k)$ is the sensor output that