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Optical solitons with differential group delay for coupled Fokas–Lenells equation using two integration schemes

Anjan Biswas [a,b,c], Yakup Yildirim [d], Emrullah Yasar [d], Qin Zhou [e,*], Mohammad F. Mahmood [f], Seithuti P. Moshokoa [c], Milivoj Belic [g]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[b] Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh 13318, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Arts and Sciences, Uludag University, 16059 Bursa, Turkey
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[f] Department of Mathematics, Howard University, Washington, DC 20059, USA
[g] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

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ABSTRACT

This paper studies differential group delay in optical fibers with coupled Fokas–Lenells equation. Two integration schemes are adopted. They are trial equation method and modified simple equation scheme. Bright, dark and singular soliton solutions are retrieved together with their respective existence criteria that are also presented.

1. Introduction

The phenomena of differential group delay (DGD) is unavoidable in soliton transmission through a fiber optic cable. This unwanted feature stems from rough handling of fibers and its nonuniformities. This cumulative effect of DGD leads to birefringent optical fibers. Mathematically, it is the vector coupled version of the model equation that is studied in this context such as coupled nonlinear Schrödinger's equation (NLSE). This paper will study the vector-coupled version of another popular model in fiber-optic transmission technology. It is the Fokas–Lenells equation (FLE) that was proposed about a decade ago and gained popularity ever since [1–15]. The coupled FLE is already studied by some authors [4,8,14,15]. There are two integration schemes that will analyze the coupled FLE in this paper. They are trial equation method and the modified simple

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

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equation scheme. These methods will reveal bright, dark and singular soliton solutions to the model. After a quick overview of these integration schemes, the details are explored in the remainder of the manuscript.

1.1. Governing model

The dimensionless form of coupled FLE is written as:

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + (c_1|q|^2 + d_1|r|^2)(\rho_1 q + i\lambda_1 q_x) + qr^*(\gamma_1 r + i\mu_1 r_x) = 0, \tag{1}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + (c_2|r|^2 + d_2|q|^2)(\rho_2 r + i\lambda_2 r_x) + rq^*(\gamma_2 q + i\mu_2 q_x) = 0. \tag{2}$$

In (1) and (2), the first term in both of these equations represents the temporal evolution of the pulses in birefringent fibers and $q(x, t)$ and $r(x, t)$ are complex valued functions that represents the soliton profiles for the two components in birefringent fibers. For $l = 1, 2$, a_l represents the group-velocity dispersion (GVD) and b_l are spatio-temporal dispersion (STD) along the two components. Next, c_l and d_l are self-phase modulation and cross-phase modulation terms respectively.

2. Revisitation of trial equation method

In this section, we revisit the main steps of trial equation algorithm:

(3) *Step 1*: Suppose a nonlinear evolution equation (NLEE) with time-dependent coefficients

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{3}$$

can be converted to an ordinary differential equation (ODE)

$$Q(U, U', U'', U''', \ldots) = 0 \tag{4}$$

using a travelling wave hypothesis $u(x, t) = U(\xi)$, $\xi = x - vt$, where $U = U(\xi)$ is an unknown function, Q is a polynomial in the variable U and its derivatives. If all terms contain derivatives, then Eq. (4) is integrated where integration constants are considered zeros.

(4) *Step 2*: Pick the trial equation

$$(U')^2 = F(U) = \sum_{l=0}^{N} \delta_l U^l \tag{5}$$

where δ_l, $(l = 0, a, \ldots, N)$ are constants to be determined. Substituting Eq. (5) and other derivative terms such as U'' or U''' and so on into Eq. (4) yields a polynomial $G(U)$ of U. According to the balance principle we can determine the value of N. Setting the coefficients of $G(U)$ to zero, we get a system of algebraic equations. Solving this system, we can determine v and values of $\delta_0, \delta_1, \ldots, \delta_N$.

(5) *Step 3*: Rewrite Eq. (5) by the integral form

$$\pm(\xi - \xi_0) = \int \frac{dU}{\sqrt{F(U)}} \tag{6}$$

Based on the discriminants of the polynomial, we classify the roots of $F(U)$, and then perform the integration in Eq. (6). Thus, we list the exact solutions to Eq. (3).

2.1. Application to FLE

In order to solve Eqs. (1) and (2) by the trial equation method, the following solution structure is hypothesized

$$q(x, t) = P_1(\xi)e^{i\phi_1(x,t)}, \tag{7}$$

$$r(x, t) = P_2(\xi)e^{i\phi_2(x,t)}, \tag{8}$$

where the wave variable ξ is given by

$$\xi = x - vt. \tag{9}$$

Here, $P_l(\xi)$ $(l = 1, 2)$ represents the amplitude component of the soliton solution and v is the speed of the soliton, while the phase component $\phi_l(x, t)$ $(l = 1, 2)$ is defined as

$$\phi_l(x, t) = -\kappa_l x + \omega_l t + \theta_l, \tag{10}$$

where κ_l are the frequencies of the solitons in each of the two components, ω_l are the wave numbers, while θ_l are the phase constants.