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Nonparaxial properties of partially coherent Lorentz-Gauss vortex beam propagating in free space

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ABSTRACT

The analytical formulae of nonparaxial propagation, paraxial propagation and far field propagation for the partially coherent Lorentz-Gauss vortex beam have been derived, the propagation properties of nonparaxial, paraxial and far field propagation are analyzed using the derived formulae. One finds that the paraxial propagation can be seen as a special case of the nonparaxial propagation at the propagation distance of the Rayleigh length, and the far field propagation can also be regarded as special case of the nonparaxial propagation at the longer propagation distance.

1. Introduction

In past years, with the development of semiconductor lasers with large divergence angles, the description of laser beam propagating in the nonparaxial regime has attracted much attention of researchers. Various methods have been developed to describe the beam propagation in the nonparaxial regime, and theory of the generalized Rayleigh-Sommmerfeld diffraction integral is a convenient method. Recently, Borghi et al. have studied the nonparaxial propagation properties of spirally polarized optical beam [1]. The vectorial nonparaxial propagation of elliptical Gaussian beam [2] and four-petal Gaussian beam [3] have been analyzed. Deng et al. have investigated the nonparaxial propagation of radially polarized light beam [4], hollow Gaussian beam [5] and rotating Cosh-Gaussian beam [6]. Kotlyar et al. have studied the nonparaxial propagation of a Gaussian optical vortex beam with initial radial polarization [7].

Zhou has investigated the nonparaxial propagation of Lorentz-Gauss beam [8,9]. Huang has investigated then nonparaxial propagation of multi-Gaussian Schell-model beam and rectangular multi-Gaussian Schell-model beam [10,11]. Li et al. have studied the nonparaxial propagation of Airy-Gaussian vortex beam [12]. Liu et al. have studied the nonparaxial properties of flat-topped vortex hollow beam [13,14], partially coherent four-petal Gaussian beam [15] and partially coherent Lorentz-Gauss beam [16]. And the nonparaxial propagation properties of the other partially coherent beam have been investigated, such as partially coherent dark hollow beam [17], partially coherent flat-topped beam [18], partially coherent anomalous hollow beam [19]. However, the nonparaxial properties of partially coherent Lorentz-Gauss vortex beam have not be reported in the past years. In this paper, the analytical formulae of nonparaxial propagation, paraxial propagation and far field propagation for the partially coherent Lorentz-Gauss vortex beam have been derived, the propagation properties are analyzed using numerical examples.

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2. Theory analysis

In the Cartesian coordinate system, the optical field of a partially coherent Lorentz-Gauss vortex beam propagating along the z axis at the source plane z = 0 can be written as [20]:

$$W(\mathbf{r}_{10},\mathbf{r}_{20},0)=E(\mathbf{r}_0,0)=\frac{w_{0x}w_{0y}}{\left(w_{0x}^2+x_{10}^2\right)\left(w_{0y}^2+y_{10}^2\right)}\exp\left(-\frac{x_{10}^2+y_{10}^2}{w_0^2}\right)\times\frac{w_{0x}w_{0y}}{\left(w_{0x}^2+x_{102}^2\right)\left(w_{0y}^2+y_{20}^2\right)}$$
$$\exp\left(\frac{x_{10}}{w_{0x}}+i\frac{y_{10}}{w_{0y}}\right)^M\exp\left(-\frac{x_{20}^2+y_{20}^2}{w_0^2}\right)\left(\frac{x_{20}}{w_{0x}}-i\frac{y_{20}}{w_{0y}}\right)^M \tag{1}$$
$$\times\exp\left[-\frac{(x_{10}-x_{20})^2}{2\sigma_x^2}-\frac{(y_{10}-y_{20})^2}{2\sigma_y^2}\right]$$

where $W(\mathbf{r}_{10},\mathbf{r}_{20},z)$ is the cross-spectral density function of partially coherent Lorentz-Gauss vortex beam at the receiver plane z; $\mathbf{r}_0=(x_0,y_0)$ is the position vectors at the source plane z = 0; w_{0x} and w_{0y} are the parameters related to the beam widths of the Lorentz part of the Lorentz-Gauss vortex along the x-axis and y-axis, respectively; w_0 is the waist width of the Gaussian part of the Lorentz-Gauss vortex beam; M is the topological charge of Lorentz-Gauss vortex beam; σ_x and σ_y are the spatial coherence length along the x-axis and y-axis, respectively.

In Eq. (1), the relationship of Lorentz function and Hermite-Gaussian function can be expressed as [21]:

$$\frac{1}{\left(x^2+w_{0x}^2\right)\left(y^2+w_{0y}^2\right)}=\frac{\pi}{2w_{0x}^2w_{0y}^2}\sum_{m=0}^{N}\sum_{n=0}^{N}\sigma_{2m}\sigma_{2n}H_{2m}\left(\frac{x}{w_{0x}}\right)H_{2n}\left(\frac{y}{w_{0y}}\right)\times\exp\left(-\frac{x^2}{2w_{0x}^2}-\frac{y^2}{2w_{0y}^2}\right) \tag{2}$$

where N is the number of the expansion and σ_{2m} and σ_{2n} are the expanded coefficients which can be found in Ref. As the even numbers $2m$ increases, the values of σ_{2m} dramatically, the N will not be large in the following numerical calculation. The 2 m order Hermite polynomial $H_{2m}(x)$ can be expressed as [22]:

$$H_{2m}(x)=\sum_{l=0}^{m}\frac{(-1)^l(2m)!}{l!(2m-2l)!}(2x)^{2m-2l} \tag{3}$$

and the vortex term of Eq. (1) can be expanded as [22]:

$$(x+iy)^M=\sum_{l=0}^{M}\frac{M!i^l}{l!(M-l)!}x^{M-l}y^l \tag{4}$$

Based on the Rayleigh-Sommerfeld diffraction integral formula [15,16], the cross-spectral density function of the partially coherent electromagnetic beams propagating along the z-axis in free space can be expressed as [15–19]:

$$W(\mathbf{r}_1,\mathbf{r}_2,z)=\left(\frac{z}{\lambda}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d\mathbf{r}_{10}d\mathbf{r}_{20}W(\mathbf{r}_{10},\mathbf{r}_{20},0)\frac{\exp\left[ik(R_2-R_1)\right]}{R_1^2R_2^2} \tag{5}$$

where $W(\mathbf{r}_1,\mathbf{r}_2,z)$ denotes the cross-spectral density function at the output plane z; $\mathbf{r}=(x,y)$ is the position vector at the receiver plane z; is the wavelength, $k=2\pi/\lambda$ is the wave number with λ being wavelength ; R_i is the distance between the source point and the point (x_i,y_i,z) and which can be written as:

$$R_i=\sqrt{(x_i-x_{i0})^2+(y_i-y_{i0})^2+z^2}\,(i=1,2) \tag{6}$$

Considering the nonparaxial propagation in free space, the R_i can be expanded into:

$$R_i=r_i+\frac{x_{i0}^2+y_{i0}^2-2x_ix_{i0}-2y_iy_{i0}}{2r_i}\,(i=1,2) \tag{7}$$

with

$$r_i=\sqrt{x_i^2+y_i^2+z^2} \tag{8}$$

Recalling the following equations [22]

$$\int_{-\infty}^{+\infty}x^n\exp\left(-px^2+2qx\right)dx=n!\exp\left(\frac{q^2}{p}\right)\left(\frac{q}{p}\right)^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{[\frac{n}{2}]}\frac{1}{k!(n-2k)!}\left(\frac{p}{4q^2}\right)^k \tag{9}$$