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Quasi-static analysis of scattering from a radially uniaxial dielectric sphere in fractional space

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ABSTRACT

Quasi-static analysis of electromagnetic scattering from a radially uniaxial (RU) sphere is presented in this paper. RU dielectric sphere is placed in an unbounded fractional dimensional space. We investigate effects of fractional dimensional space on behavior of potential distribution due to the RU sphere. For this purpose, numerical results of electric potential for different dimensions of the fractional space have been presented and discussed.

1. Introduction

Fractional Calculus (FC) is a branch of mathematics that deals with situations for which the order of the derivative/integral operators to arbitrary value (real and even complex) can be evaluated [1,2]. Particularly, owing to nonlocality of the fractional order operators, these are very helpful in modeling physical problems in which memory is involved [3–10]. FC had been applied to various problems in electromagnetics and interesting results have been obtained which emphasize certain distinguished and notable features of these operators [11–15]. Inspired by the study of FC, some other operators are also fractionalized. Fractional curl operator was proposed and its applications to electromagnetic problems had been discussed in [16,17].

To solve problems in which fractional dimensional space (FDS) is involved FC may be used [18–25]. Mathematical formulation of Lagrangian and Hamiltonian formalism of dynamics and electromagnetic fields in FDS was investigated in [26,27], and mathematical representation of electromagnetic waves in dielectric material using FDS is reported in [11,25]. Solutions of the Helmholtz's equation in FDS were derived using separation of variable method [28–31]. Knowledge of Green's function is very basic in the study of radiation and scattering problems. Radiation from canonical source buried under dielectric half-space geometry was derived by Abbas et al. [32–34]. Definition of multipoles in FDS was studied by Muslih et al. in [35]. The solutions of fractional Laplace's and Poisson's equations describing the solutions of the electrostatic problems in FDS have been derived [32–36].

The intermediate solution of complete set of Maxwell equations and static analysis in electromagnetic theory is named as quasi-static analysis (QSA). QSA can be employed if size of the object/scatterer is much smaller compared to operating wavelength of the applied electromagnetic field [37,38]. Quasi-static model is developed to investigate ultrasonic scattering from imperfect interfaces [39]. QSA of electromagnetic scattering from a chiral sphere placed in free space and chiral sphere placed in chiral material are available in [40]. RU material is anisotropic material having permittivity and/or permeability in

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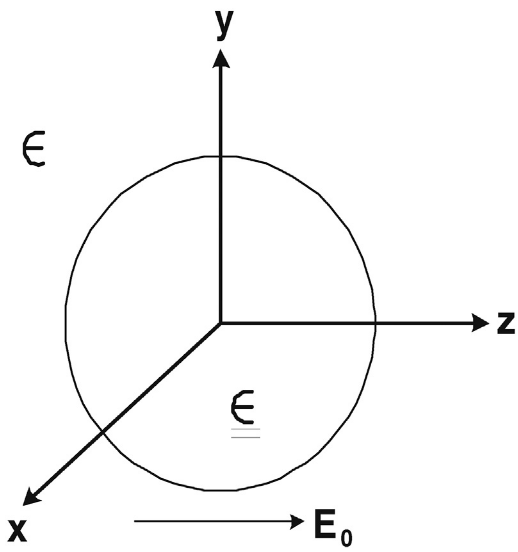

Fig. 1. Anisotropic dielectric sphere placed in fractional dimensional space (FDS).

radial direction different from other two directions [41]. Due to applications of RU structures in the designing of metamaterial and specially in electromagnetic cloaking, RU structures/geometries have been widely addressed in literature in the recent few years [42–48].

In the present paper, we have investigated electromagnetic scattering form RU sphere which is placed in FDS and potentials/field distribution inside and outside the sphere owing to the effective presence of fractional parameter in these expressions is evaluated. Mathematical formulation of the problem is given in Section 2. In Section 3 numerical results are presented. Finally, Section 4 contains concluding remarks.

2. Mathematical formulation

Consider a radially uniaxial dielectric sphere having radius a placed in an unbounded isotropic dielectric medium. It has been assumed that medium occupying the sphere is of integer dimensional space whereas the medium hosting the sphere is NID space. The permittivity of host medium outside the sphere is denoted by ϵ, whereas permittivity of medium inside the sphere is represented by a tensor given below

$$\bar{\bar{\varepsilon}} = \varepsilon_o[\varepsilon_{rad}\mathbf{u}_r\mathbf{u}_r + \varepsilon_{\tan}(\mathbf{u}_\theta\mathbf{u}_\theta + \mathbf{u}_\phi\mathbf{u}_\phi)] \tag{1}$$

where ε_{rad} and ε_{tan} are electrical responses in radial and tangential directions respectively and $\mathbf{u_i}$ are unit vectors in directions for the spherical coordinate system.

External applied field under the quasi-static approximation may be written as [35]

$$\boldsymbol{E}_{(\alpha,inc)} = \hat{z}(\alpha-2)E_o \tag{2}$$

where α is the fractional parameter. As it is also assumed that size of sphere is very small as compared to the operating wavelength, this means that wave number for the host medium, that is, $k \longrightarrow 0$ (Fig. 1).

Corresponding scalar static electric potential is

$$\phi_{inc} = -E_o(\alpha-2)r\ \cos\ \theta \tag{3}$$

The electrostatic potential in an anisotropic medium must satisfy the Laplace equation which is written below

$$\nabla\cdot(\bar{\bar{\varepsilon}}\cdot\nabla\phi) = 0 \tag{4}$$

Solutions of the above equation can be obtained using the separation of variables method. This method allows us to write

$$\phi(r,\theta) = R(r)\Theta(\theta)$$

Expression for each function is written below

$$R(r) = Ar^{\nu} + Br^{-\nu-1} \tag{5}$$

$$\Theta(\theta) = CP_n^m(\cos\ \theta) + DQ_n^m(\cos\ \theta) \tag{6}$$