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Resonant optical solitons with dual-power law nonlinearity and fractional temporal evolution

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ABSTRACT

This paper secures dark and singular resonant optical solitons that is studied with dual-power law nonlinearity and fractional temporal evolution. Khalil's conformable fractional derivative is put into perspective to retrieve these soliton solutions. The parameter restrictions for the existence of such solitons are also indicated.

1. Introduction

One can only paint a better and improved picture of telecommunications industry with the aid of soliton molecules. These are bit carriers of information across inter-continental distances all around the globe. The governing model is the nonlinear Schrödinger's equation (NLSE) that is studied with varied forms of dispersion and a variety of nonlinear structures apart from Kerr form. This paper studies the resonant NLSE that is being considered with dual-power law nonlinearity and fractional temporal evolution of pulses. The results of this manuscript will thus carry unprecendented novelty. The

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evolution of pulses can be therefore controlled and this technology enables the mitigation of a growing problem in soliton transmission technology, namely the Internet bottleneck. There are several mathematical mechanisms that can handle this dynamics analytically [1–15]. This paper will employ Khalil's concept of fractional derivative to retrieve dark and singular soliton solutions to the resonant NLSE. This derivative was first introduced in 2014 and has gained popularity ever since [13]. The rest of the paper is dedicated to the derivation of the soliton solutions after a quick overview of the basics.

1.1. Mathematical model

The governing resonant NLSE with perturbation terms and fractional temporal evolution that is studied in nonlinear optics is formulated in its dimensionless form as: [5,7]:

$$\begin{aligned} & iq_t^{\mu} + aq_{xx} + b\frac{|q|_{xx}}{|q|}q + \left(c_1|q|^{2n} + c_2|q|^{4n}\right)q \\ & = i\left\{\delta q_x + \lambda\left(|q|^{2n}q\right)_x + \nu\left(|q|^{2n}\right)_x q + \theta|q|^{2n}q_x\right\} + \sigma\frac{q_{xx}^*}{|q|^2}q^2, \end{aligned} \tag{1}$$

where D_t^{μ} is the conformable derivative operator of order $\mu \in (0, \quad 1]$ in the $t > 0$ is defined by the following [6]

$$D_t^{\mu}(f)(t) = \lim_{\varepsilon \to 0}\frac{f(t+\varepsilon t^{1-\mu}) - f(t)}{\varepsilon}, \qquad f : (0, \infty) \longrightarrow R. \tag{2}$$

Later on, many effective methods for obtaining exact traveling wave solutions of various nonlinear fractional evolution equations via this fractional derivative have been presented [1,13,14].

Now, in (1), the first term represents the fractional temporal evolution where $0 < \mu \le 1$, and b is the coefficient of group velocity dispersion (GVD) while the two nonlinear forms are the coefficients of c_1 and c_2 that bind together and balance with GVD in a delicate manner for sustaining these solitons. On the right hand side δ accounts for inter-modal dispersion and λ is from self-steepening term, while ν and θ are from nonlinear dispersions. The coefficient of σ is from Madeling fluids that makes the model resonant. The full nonlinearity parameter is n. The detailed analysis of the model is discussed in the subsequent section.

2. Mathematical analysis

In order to solve Eq. (1), we use the wave transformation

$$q(x, t) = g(s)e^{i\Phi(x,t)} \tag{3}$$

where $g(s)$ represents the shape of the pulse and

$$s = x + \left(\frac{\delta + 2(a+\sigma)\kappa}{\mu}\right)t^{\mu}, \tag{4}$$

and the phase component is defined as

$$\Phi(x, t) = -\kappa x + \frac{\omega}{\mu}t^{\mu} + \theta_0. \tag{5}$$

Substituting (3) into (1), we obtain an ordinary differential equation (ODE)

$$(a + b - \sigma)g'' - (\omega + \delta\kappa + a\kappa^2 - \sigma\kappa^2)g + \left(c_1 - \lambda\kappa - \theta\kappa\right)g^{2n+1} + c_2 g^{4n+1} = 0, \tag{6}$$

along with the constraint condition on the perturbation parameters

$$(2n + 1)\lambda + 2n\nu + \theta = 0. \tag{7}$$

In order to obtain closed form solutions, we use the transformation

$$g = U^{1/2n}, \tag{8}$$

that will reduce Eq. (6) into the ODE

$$\begin{aligned} & (a + b - \sigma)\left\{2nUU'' + (1 - 2n)(U')^2\right\} - 4n^2(\omega + \delta\kappa + a\kappa^2 - \sigma\kappa^2)U^2 \\ & +4n^2\left(c_1 - \lambda\kappa - \theta\kappa\right)U^3 + 4n^2 c_2 U^4 = 0. \end{aligned} \tag{9}$$

Balancing the terms UU'' and U^4 in Eq. (9) yields $N = 1$. In this section, we construct some exact traveling wave solutions of Eq. (1) by using the modified simple equation method [8].

Suppose that we have the formal solution of Eq. (9) as follows

$$U(s) = A_0 + A_1\left(\frac{\phi'}{\phi}\right), \tag{10}$$